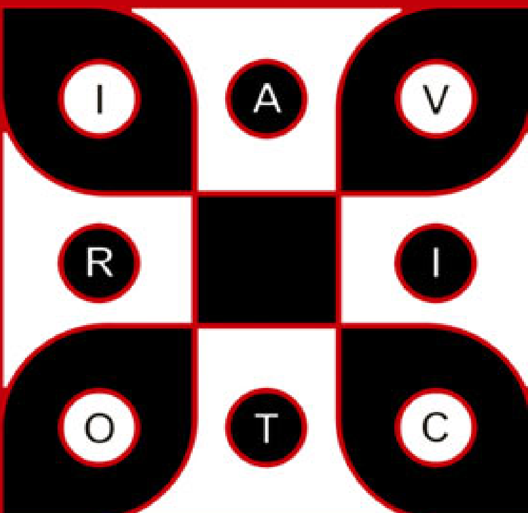
Good old-fashioned

Challenging Puzzles

and perplexing mathematical problems



H. E. Dudeney

GOOD OLD-FASHIONED CHALLENGING PUZZLES
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Summersdale Publishers Ltd
46 West Street
Chichester
West Sussex
PO19 1RP
UK

www.summersdale.com

Printed and bound in Great Britain.

ISBN: 1-84024-557-3
ISBN 13: 978-1-84024-557-8

EDITOR'S NOTE

Henry Ernest Dudeney (1857–1930) was an author and mathematician, and is known as one of the country's foremost creators of puzzles. His puzzles regularly appeared in newspapers and magazines.

Good Old-fashioned Challenging Puzzles is a selection of mathematical brain-teasers from his book *Amusements in Mathematics*, first published in 1917 and hailed by *The Spectator* as 'not only an amusement but a revelation'. Some of the problems are, as Dudeney admitted, 'not unworthy of the attention of the advanced mathematician'.

For today's lovers of sudoku, chess, poker, bridge, cryptic crosswords and other brain-training games, these good old-fashioned puzzles and problems will provide many hours of amusement and an unparalleled opportunity to exercise your logical and mathematical agility. We have chosen to leave Dudeney's uniquely witty preface and introductions largely as they were when the book first appeared.

PREFACE

In issuing this volume of my Mathematical Puzzles, of which some have appeared in periodicals and others are given here for the first time, I must acknowledge the encouragement that I have received from many unknown correspondents, at home and abroad, who have expressed a desire to have the problems in a collected form, with some of the solutions given at greater length than is possible in magazines and newspapers. Though I have included a few old puzzles that have interested the world for generations, where I felt that there was something new to be said about them, the problems are in the main original.

On the question of Mathematical Puzzles in general there is, perhaps, little more to be said than I have written elsewhere. The history of the subject entails nothing short of the actual story of the beginnings and development of exact thinking in man. The historian must start from the time when man first succeeded in counting his ten fingers and in dividing an apple into two approximately equal parts. Every puzzle that is worthy of consideration can be referred to mathematics and logic. Every man, woman, and child who tries to 'reason out' the answer to the simplest puzzle is working, though not of necessity consciously, on mathematical lines. Even those puzzles that we have no way of attacking except by haphazard attempts can be brought under a method of what has been called 'glorified trial'—a system of shortening our labours by avoiding or eliminating what our reason tells us is useless. It is, in fact, not easy to say sometimes where the 'empirical' begins and where it ends.

When a man says, 'I have never solved a puzzle in my life,' it is difficult to know exactly what he means, for every intelligent individual is doing it every day. The unfortunate inmates of our lunatic asylums are sent there expressly because they cannot solve puzzles—because they have lost their powers of reason. If there were no puzzles to solve, there would be no questions to ask; and if there were no questions to be asked, what a world it would be! We should all be equally omniscient, and conversation would be useless and idle.

It is possible that some few exceedingly sober-minded mathematicians, who are impatient of any terminology in their favourite science but the academic, and who object to the elusive *x* and *y* appearing under any other names, will have wished that various problems had been presented in a less popular dress and introduced with a less flippant phraseology. I can only refer them to the first word of my title and remind them that we are primarily out to be amused—not, it is true, without some hope of picking up morsels of knowledge by the way. If the manner is light, I can only say, in the words of Touchstone, that it is 'an ill-favoured thing, sir, but my own; a poor humour of mine, sir.'

As for the question of difficulty, some of the puzzles, especially in the Arithmetical and Algebraical category, are quite easy. Yet some of those examples that look the simplest should not be passed over without a little consideration, for now and again it will be found that there is some more or less subtle pitfall or trap into which the reader may be apt to fall. It is good exercise to cultivate the habit of being very wary over the exact wording of a puzzle. It teaches exactitude and caution. But some of the problems are very hard nuts indeed, and not unworthy of the attention of the advanced mathematician. Readers will doubtless select according to their individual tastes.

In many cases only the mere answers are given. This leaves the beginner something to do on his own behalf in working out the method of solution, and saves space that would be wasted from the point of view of the advanced student. On the other hand, in particular cases where it seemed likely to interest, I have given rather extensive solutions and treated problems in a general manner. It will often be found that the notes on one problem will serve to elucidate a good many others in the book; so that the reader's difficulties will sometimes be found cleared up as he advances. Where it is possible to say a thing in a manner that may be 'understanded of the people' generally, I prefer to use this simple phraseology, and so engage the attention and interest of a larger public. The mathematician will in such cases have no difficulty in expressing the matter under consideration in terms of his familiar symbols.

I have taken the greatest care in reading the proofs, and trust that any errors that may have crept in are very few. If any such should occur, I can only plead, in the words of Horace, that 'good Homer sometimes nods,' or, as the bishop put it, 'Not even the youngest curate in my diocese is infallible.'

I have to express my thanks in particular to the proprietors of *The Strand Magazine*, *Cassell's Magazine*, *The Queen*, *Tit-Bits*, and *The Weekly Dispatch* for their courtesy in allowing me to reprint some of the puzzles that have appeared in their pages.

THE AUTHORS' CLUB
March 25, 1917

CONTENTS

ARITHMETICAL AND ALGEBRAICAL PROBLEMS

'And what was he?
Forsooth, a great arithmetician.'

Othello, I. i.

The puzzles in this department are roughly thrown together in classes for the convenience of the reader. Some are very easy, others quite difficult. But they are not arranged in any order of difficulty—and this is intentional, for it is well that the solver should not be warned that a puzzle is just what it seems to be. It may, therefore, prove to be quite as simple as it looks, or it may contain some pitfall into which, through want of care or over-confidence, we may stumble.

Also, the arithmetical and algebraical puzzles are not separated in the manner adopted by some authors, who arbitrarily require certain problems to be solved by one method or the other. The reader is left to make his own choice and determine which puzzles are capable of being solved by him on purely arithmetical lines.

MONEY PUZZLES

'Put not your trust in money, but put your money in trust.'

OLIVER WENDELL HOLMES.

1. AT A CATTLE MARKET.

Three countrymen met at a cattle market. 'Look here,' said Hodge to Jakes, 'I'll give you six of my pigs for one of your horses, and then you'll have twice as many animals here as I've got.' 'If that's your way of doing business,' said Durrant to Hodge, 'I'll give you fourteen of my sheep for a horse, and then you'll have three times as many animals as I.' 'Well, I'll go better than that,' said Jakes to Durrant; 'I'll give you four cows for a horse, and then you'll have six times as many animals as I've got here.'

No doubt this was a very primitive way of bartering animals, but it is an interesting little puzzle to discover just how many animals Jakes, Hodge, and Durrant must have taken to the cattle market.

2. THE TWO AEROPLANES.

A man recently bought two aeroplanes, but afterwards found that they would not answer the purpose for which he wanted them. So he sold them for £600 each, making a loss of 20 per cent on one machine and a profit of 20 per cent on the other. Did he make a profit on the whole transaction, or a loss? And how much?

3. THE MILLIONAIRE'S PERPLEXITY.

Mr. Morgan G. Bloomgarten, the millionaire, known in the States as the Clam King, had, for his sins, more money than he knew what to do with. It bored him. So he determined to persecute some of his poor but happy friends with it. They had never done him any harm, but he resolved to inoculate them with the 'source of all evil.' He therefore proposed to distribute a million dollars among them and watch them go rapidly to the bad. But he was a man of strange fancies and superstitions, and it was an inviolable rule with him never to make a gift that was not either one dollar or some power of seven—such as 7, 49, 343, 2,401, which numbers of dollars are produced by simply multiplying sevens together. Another rule of his was that he would never give more than six persons exactly the same sum. Now, how was he to distribute the 1,000,000 dollars? You may distribute the money among as many people as you like, under the conditions given.

4. THE GROCER AND DRAPER.

A country 'grocer and draper' had two rival assistants, who prided themselves on their rapidity in serving customers. The young man on the grocery side could weigh up two one-pound parcels of sugar per minute, while the drapery assistant could cut three one-yard lengths of cloth in the same time. Their employer, one slack day, set them a race, giving the grocer a barrel of sugar and telling him to weigh up forty-eight one-pound parcels of sugar while the draper divided a roll of forty-eight yards of cloth into yard pieces. The two men were interrupted together by customers for nine minutes, but the draper was disturbed seventeen times as long as the grocer. What was the result of the race?

5. JUDKINS'S CATTLE.

Hiram B. Judkins, a cattle-dealer of Texas, had five droves of animals, consisting of oxen, pigs, and sheep, with the same number of animals in each drove. One morning he sold all that he had to eight dealers. Each dealer bought the same number of animals, paying seventeen dollars for each ox, four dollars for each pig, and two dollars for each sheep; and Hiram received in all three hundred and one dollars. What is the greatest number of animals he could have had? And how many would there be of each kind?

AGE AND KINSHIP PUZZLES

'The days of our years are threescore years and ten.'
—*Psalm* xc. 10.

For centuries it has been a favourite method of propounding arithmetical puzzles to pose them in the form of questions as to the age of an individual. They generally lend themselves to very easy solution by the use of algebra, though often the difficulty lies in stating them correctly. They may be made very complex and may demand considerable ingenuity, but no general laws can well be laid down for their solution. The solver must use his own sagacity. As for puzzles in relationship or kinship, it is quite curious how bewildering many people find these things. Even in ordinary conversation, some statement as to relationship, which is quite clear in the mind of the speaker, will immediately tie the brains of other people into knots. In such cases the best course is to sketch a brief genealogical table, when the eye comes immediately to the assistance of the brain. In these days, when we have a growing lack of respect for pedigrees, most people have got out of the habit of rapidly drawing such tables, which is to be regretted, as they would save a lot of time and brain racking on occasions.

6. MAMMA'S AGE.

Tommy: 'How old are you, mamma?'
Mamma: 'Let me think, Tommy. Well, our three ages add up to exactly seventy years.'
Tommy: 'That's a lot, isn't it? And how old are you, papa?'
Papa: 'Just six times as old as you, my son.'
Tommy: 'Shall I ever be half as old as you, papa?'
Papa: 'Yes, Tommy; and when that happens our three ages will add up to exactly twice as much as to-day.'
Tommy: 'And supposing I was born before you, papa; and supposing mamma had forgot all about it, and hadn't been at home when I came;
and supposing—'
Mamma: 'Supposing, Tommy, we talk about bed. Come along, darling. You'll have a headache.'

Now, if Tommy had been some years older he might have calculated the exact ages of his parents from the information they had given him. Can you find out the exact age of mamma?

7. THEIR AGES.

'My husband's age,' remarked a lady the other day, 'is represented by the figures of my own age reversed. He is my senior, and the difference between our ages is one-eleventh of their sum.'

8. ROVER'S AGE.

'Now, then, Tommy, how old is Rover?' Mildred's young man asked her brother.

'Well, five years ago,' was the youngster's reply, 'sister was four times older than the dog, but now she is only three times as old.'

Can you tell Rover's age?

9. CONCERNING TOMMY'S AGE.

Tommy Smart was recently sent to a new school. On the first day of his arrival the teacher asked him his age, and this was his curious reply: 'Well, you see, it is like this. At the time I was born—I forget the year—my only sister, Ann, happened to be just one-quarter the age of mother, and she is now one-third the age of father.' 'That's all very well,' said the teacher, 'but what I want is not the age of your sister Ann, but your own age.' 'I was just coming to that,' Tommy answered; 'I am just a quarter of mother's present age, and in four years' time I shall be a quarter the age of father. Isn't that funny?'

This was all the information that the teacher could get out of Tommy Smart. Could you have told, from these facts, what was his precise age? It is certainly a little puzzling.

10. THE BAG OF NUTS.

Three boys were given a bag of nuts as a Christmas present, and it was agreed that they should be divided in proportion to their ages, which together amounted to 17½ years. Now the bag contained 770 nuts, and as often as Herbert took four Robert took three, and as often as Herbert took six Christopher took seven. The puzzle is to find out how many nuts each had, and what were the boys' respective ages.

11. A FAMILY PARTY.

A certain family party consisted of 1 grandfather, 1 grandmother, 2 fathers, 2 mothers, 4 children, 3 grandchildren, 1 brother, 2 sisters, 2 sons, 2 daughters, 1 father-in-law, 1 mother-in-law, and 1 daughter-in-law. Twenty-three people, you will say. No; there were only seven persons present. Can you show how this might be?

CLOCK PUZZLES

'Look at the clock!'

Ingoldsby Legends.

In considering a few puzzles concerning clocks and watches, and the times recorded by their hands under given conditions, it is well that a particular convention should always be kept in mind. It is frequently the case that a solution requires the assumption that the hands can actually record a time involving a minute fraction of a second. Such a time, of course, cannot be really indicated. Is the puzzle, therefore, impossible of solution? The conclusion deduced from a logical syllogism depends for its truth on the two premises assumed, and it is the same in mathematics. Certain things are antecedently assumed, and the answer depends entirely on the truth of those assumptions.

'If two horses,' says Lagrange, 'can pull a load of a certain weight, it is natural to suppose that four horses could pull a load of double that weight, six horses a load of three times that weight. Yet, strictly speaking, such is not the case. For the inference is based on the assumption that the four horses pull alike in amount and direction, which in practice can scarcely ever be the case. It so happens that we are frequently led in our reckonings to results which diverge widely from reality. But the fault is not the fault of mathematics; for mathematics always gives back to us exactly what we have put into it. The ratio was constant according to that supposition. The result is founded upon that supposition. If the supposition is false the result is necessarily false.'

If one man can reap a field in six days, we say two men will reap it in three days, and three men will do the work in two days. We here assume, as in the case of Lagrange's horses, that all the men are exactly equally capable of work. But we assume even more than this. For when three men get together they may waste time in gossip or play; or, on the other hand, a spirit of rivalry may spur them on to greater diligence. We may assume any conditions we like in a problem, provided they be clearly expressed and understood, and the answer will be in accordance with those conditions.

12. A TIME PUZZLE.

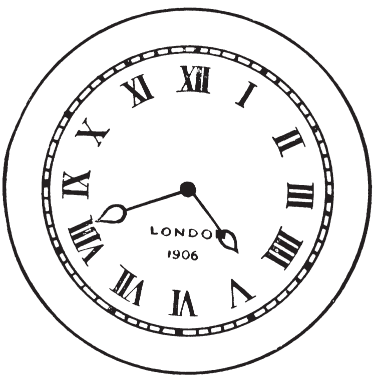
How many minutes is it until six o'clock if fifty minutes ago it was four times as many minutes past three o'clock?

13. THE WAPSHAW'S WHARF MYSTERY.

There was a great commotion in Lower Thames Street on the morning of January 12, 1887. When the early members of the staff arrived at Wapshaw's Wharf they found that the safe had been broken open, a considerable sum of money removed, and the offices left in great disorder. The night watchman was nowhere to be found, but nobody who had been acquainted with him for one moment suspected him to be guilty of the robbery. In this belief the proprietors were confirmed when, later in the day, they were informed that the poor fellow's body had been picked up by the River Police. Certain marks of violence pointed to the fact that he had been brutally attacked and thrown into the river. A watch found in his pocket had stopped, and this was a valuable clue to the time of the outrage. But a very stupid officer had actually amused himself by turning the hands round and round, trying to set the watch going again. After he had been severely reprimanded for this serious indiscretion, he was asked whether he could remember the time that was indicated by the watch when found. He replied that he could not, but he recollected that the hour hand and minute hand were exactly together, one above the other, and the second hand had just passed the forty-ninth second. More than this he could not remember.

What was the exact time at which the watchman's watch stopped? The watch is, of course, assumed to have been an accurate one.

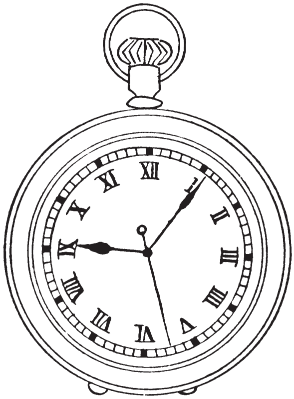
14. CHANGING PLACES.



The clock face indicates a little before 42 minutes past 4. The hands will again point at exactly the same spots a little after 23 minutes past 8. In fact, the hands will have changed places. How many times do the hands of a clock change places between three o'clock p.m. and midnight? And out of all the pairs of times indicated by these changes, what is the exact time when the minute hand will be nearest to the point IX?

15. THE STOP-WATCH.

We have here a stop-watch with three hands. The second hand, which travels once round the face in a minute, is the one with the little ring at its end near the centre. Our dial indicates the



exact time when its owner stopped the watch. You will notice that the three hands are nearly equidistant. The hour and minute hands point to spots that are exactly a third of the circumference apart, but the second hand is a little too advanced. An exact equidistance for the three hands is not possible. Now, we want to know what the time will be when the three hands are next at exactly the same distances as shown from one another. Can you state the time?

16. THE THREE CLOCKS.

On Friday, April 1, 1898, three new clocks were all set going precisely at the same time—twelve noon. At noon on the following day it was found that clock A had kept perfect time, that clock B had gained exactly one minute, and that clock C had lost exactly one minute. Now, supposing that the clocks B and C had not been regulated, but all three allowed to go on as they had begun, and that they maintained the same rates of progress without stopping, on what date and at what time of day would all three pairs of hands again point at the same moment at twelve o'clock?

17. THE VILLAGE SIMPLETON.

A facetious individual who was taking a long walk in the country came upon a yokel sitting on a stile. As the gentleman was not quite sure of his road, he thought he would make inquiries of the local inhabitant; but at the first glance he jumped too hastily to the conclusion that he had dropped on the village idiot. He therefore decided to test the fellow's intelligence by first putting to him the simplest question he could think of, which was, 'What day of the week is this, my good man?' The following is the smart answer that he received:—

'When the day after to-morrow is yesterday, to-day will be as far from Sunday as to-day was from Sunday when the day before yesterday was to-morrow.'

Can the reader say what day of the week it was? It is pretty evident that the countryman was not such a fool as he looked. The gentleman went on his road a puzzled but a wiser man.

LOCOMOTION AND SPEED PUZZLES

'The race is not to the swift.'

—*Ecclesiastes* ix. II.

18. THE TWO TRAINS.

Two trains start at the same time, one from London to Liverpool, the other from Liverpool to London. If they arrive at their destinations one hour and four hours respectively after passing one another, how much faster is one train running than the other?

19. THE THREE VILLAGES.

I set out the other day to ride in a motor-car from Acrefield to Butterford, but by mistake I took the road going via Cheesebury, which is nearer Acrefield than Butterford, and is twelve miles to the left of the direct road I should have travelled. After arriving at Butterford I found that I had gone thirty-five miles. What are the three distances between these villages, each being a whole number of miles?

20. DONKEY RIDING.

During a visit to the seaside Tommy and Evangeline insisted on having a donkey race over the mile course on the sands. Mr.

Dobson and some of his friends whom he had met on the beach acted as judges, but, as the donkeys were familiar acquaintances and declined to part company the whole way, a dead heat was unavoidable. However, the judges, being stationed at different points on the course, which was marked off in quarter-miles, noted the following results:–The first three-quarters were run in six and three-quarter minutes, the first half-mile took the same time as the second half, and the third quarter was run in exactly the same time as the last quarter. From these results Mr. Dobson amused himself in discovering just how long it took those two donkeys to run the whole mile. Can you give the answer?

DIGITAL PUZZLES

'Nine worthies were they called.'
DRYDEN: *The Flower and the Leaf.*

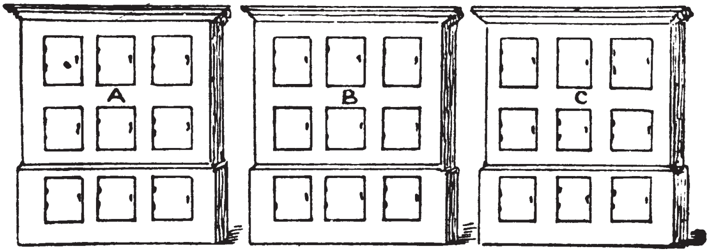
I give these puzzles, dealing with the nine digits, a class to themselves, because I have always thought that they deserve more consideration than they usually receive. Beyond the mere trick of 'casting out nines,' very little seems to be generally known of the laws involved in these problems, and yet an acquaintance with the properties of the digits often supplies, among other uses, a certain number of arithmetical checks that are of real value in the saving of labour. Let me give just one example—the first that occurs to me.

If the reader were required to determine whether or not 15,763,530,163,289 is a square number, how would he proceed? If the number had ended with a 2, 3, 7, or 8 in the digits place, of course he would know that it could not be a square, but there is nothing in its apparent form to prevent its being one. I suspect that in such a case he would set to work, with a sigh or a groan, at the laborious task of extracting the square root. Yet if he had given a little attention to the study of the digital properties of numbers, he would settle the question in this simple way. The sum of the digits is 59, the sum of which is 14, the sum of which is 5 (which I call the 'digital root'), and therefore I know that the number cannot be a square, and for this reason. The digital root of successive square numbers from 1 upwards is always 1, 4, 7, or 9, and can never be anything else. In fact, the series, 1, 4, 9, 7, 7, 9, 4, 1, 9, is repeated into infinity. The analogous series for triangular numbers is 1, 3, 6, 1, 6, 3, 1, 9, 9. So here we have a similar negative check, for a number cannot be triangular (that is, $\frac{n^2 + n}{2}$) if its digital root be 2, 4, 5, 7, or 8.

21. THE BARREL OF BEER.

A man bought an odd lot of wine in barrels and one barrel containing beer. These are shown in the illustration, marked with the number of gallons that each barrel contained. He sold a quantity of the wine to one man and twice the quantity to another, but kept the beer to himself. The puzzle is to point out which barrel contains beer. Can you say which one it is? Of course, the man sold the barrels just as he bought them, without manipulating in any way the contents.

22. THE LOCKERS PUZZLE.

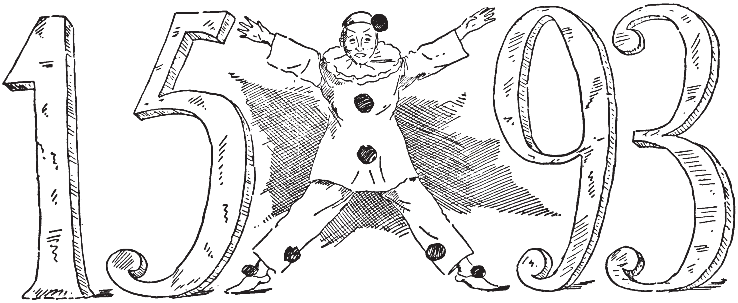


A man had in his office three cupboards, each containing nine lockers, as shown in the diagram. He told his clerk to place a different one-figure number on each locker of cupboard A, and to do the same in the case of B, and of C. As we are here allowed to call nought a digit, and he was not prohibited from using nought as a number, he clearly had the option of omitting any one of ten digits from each cupboard. Now, the employer did not say the lockers were to be numbered in any numerical order, and he was surprised to find, when the work was done, that the figures had apparently been mixed up indiscriminately. Calling upon his clerk for an explanation, the eccentric lad stated that the notion had occurred to him so to arrange the figures that in each case they formed a simple addition sum, the two upper rows of figures producing the sum in the lowest row. But the most surprising point was this: that he had so arranged them that the addition in A gave the smallest possible sum, that the addition in C gave the largest possible sum, and that all the nine digits in the three totals were different. The puzzle is to show how this could be done. No decimals are allowed and the nought may not appear in the hundreds place.

23. THE THREE GROUPS.

There appeared in 'Nouvelles Annales de Mathématiques' the following puzzle as a modification of one of my 'Canterbury Puzzles.' Arrange the nine digits in three groups of two, three, and four digits, so that the first two numbers when multiplied together make the third. Thus, 12 × 483 = 5,796. I now also propose to include the cases where there are one, four, and four digits, such as 4 × 1,738 = 6,952. Can you find all the possible solutions in both cases?

24. THE PIERROT'S PUZZLE.



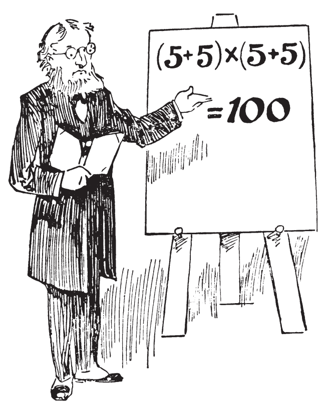
The Pierrot in the illustration is standing in a posture that represents the sign of multiplication. He is indicating the peculiar fact that 15 multiplied by 93 produces exactly the same figures (1,395), differently arranged. The puzzle is to take any four digits you like (all different) and similarly arrange them so that the number formed on one side of the Pierrot when multiplied by the number on the other side shall produce the same figures. There are very few ways of doing it, and I shall give all the cases possible. Can you find them all? You are allowed to put two figures on each side of the Pierrot as in the example shown, or to place a single figure on one side and three figures on the other. If we only used three digits instead of four, the only possible ways are these: 3 multiplied by 51 equals 153, and 6 multiplied by 21 equals 126.

25. THE CAB NUMBERS.

A London policeman one night saw two cabs drive off in opposite directions under suspicious circumstances. This officer was a particularly careful and wide-awake man, and he took out his pocket-book to make an entry of the numbers of the cabs, but discovered that he had lost his pencil. Luckily, however, he found a small piece of chalk, with which he marked the two numbers on the gateway of a wharf close by. When he returned to the same spot on his beat he stood and looked again at the numbers, and noticed this peculiarity, that all the nine digits (no nought) were used and that no figure was repeated, but that if he multiplied the two numbers together they again produced the nine digits, all once, and once only. When one of the clerks arrived at the wharf in the early morning, he observed the chalk marks and carefully rubbed them out. As the policeman could not remember them, certain mathematicians were then consulted as to whether there was any known method for discovering all the pairs of numbers that have the peculiarity that the officer had noticed; but they knew of none. The investigation, however, was interesting, and the following question out of many was proposed: What two numbers, containing together all the nine digits, will, when multiplied together, produce another number (the highest possible) containing also all the nine digits? The nought is not allowed anywhere.

26. THE FOUR SEVENS.

In the illustration Professor Rackbrane is seen demonstrating one of the little posers with which he is accustomed to entertain his class. He believes that by taking his pupils off the beaten tracks he is the better able to secure their attention, and to induce original and ingenious



methods of thought. He has, it will be seen, just shown how four 5's may be written with simple arithmetical signs so as to represent 100. Every juvenile reader will see at a glance that his example is quite correct. Now, what he wants you to do is this: Arrange four 7's (neither more nor less) with arithmetical signs so that they shall represent 100. If he had said we were to use four 9's we might at once have written $99\frac{9}{9}$, but the four 7's call for rather more ingenuity. Can you discover the little trick?

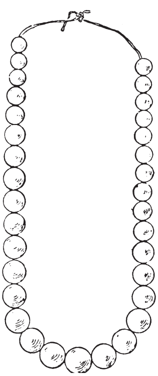
VARIOUS ARITHMETICAL AND ALGEBRAICAL PROBLEMS

'Variety's the very spice of life,
That gives it all its flavour.'

COWPER: *The Task*.

27. THE THIRTY-THREE PEARLS.

'A man I know,' said Teddy Nicholson at a certain family party,



'possesses a string of thirty-three pearls. The middle pearl is the largest and best of all, and the others are so selected and arranged that, starting from one end, each successive pearl is worth £100 more than the preceding one, right up to the big pearl. From the other end the pearls increase in value by £150 up to the large pearl. The whole string is worth £65,000. What is the value of that large pearl?'

'Pearls and other articles of clothing,' said Uncle Walter, when the price of the precious gem had been discovered, 'remind me of Adam and Eve. Authorities, you may not know, differ as to the number of apples that were eaten by Adam and Eve. It is the opinion of some that Eve 8 (ate) and Adam 2 (too), a total of 10 only. But certain mathematicians have figured it out differently, and hold that Eve 8 and Adam a total of 16. Yet the most recent investigators think the above figures entirely wrong, for if Eve 8 and Adam 8 2, the total must be 90.'

'Well,' said Harry, 'it seems to me that if there were giants in those days, probably Eve 8 1 and Adam 8 2, which would give a total of 163.'

'I am not at all satisfied,' said Maud. 'It seems to me that if Eve 8 1 and Adam 8 1 2, they together consumed 893.'

'I am sure you are all wrong,' insisted Mr. Wilson, 'for I consider that Eve 8 1 4 Adam, and Adam 8 1 2 4 Eve, so we get a total of 8,938.'

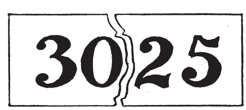
'But, look here,' broke in Herbert. 'If Eve 8 1 4 Adam and Adam 8 1 2 4 2 oblige Eve, surely the total must have been 82,056!'

At this point Uncle Walter suggested that they might let the matter rest. He declared it to be clearly what mathematicians call an indeterminate problem.

28. PAINTING THE LAMP-POSTS.

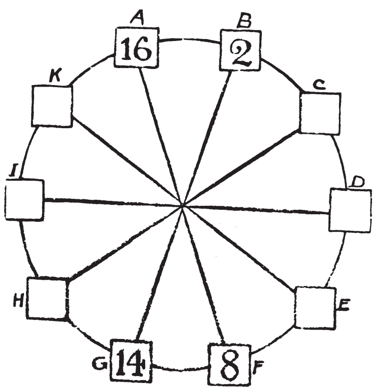
Tim Murphy and Pat Donovan were engaged by the local authorities to paint the lamp-posts in a certain street. Tim, who was an early riser, arrived first on the job, and had painted three on the south side when Pat turned up and pointed out that Tim's contract was for the north side. So Tim started afresh on the north side and Pat continued on the south. When Pat had finished his side he went across the street and painted six posts for Tim, and then the job was finished. As there was an equal number of lamp-posts on each side of the street, the simple question is: Which man painted the more lamp-posts, and just how many more?

29. THE TORN NUMBER.



I had the other day in my possession a label bearing the number 3 0 2 5 in large figures. This got accidentally torn in half, so that 3 0 was on one piece and 2 5 on the other, as shown on the illustration. On looking at these pieces I began to make a calculation, scarcely conscious of what I was doing, when I discovered this little peculiarity. If we add the 3 0 and the 2 5 together and square the sum we get as the result the complete original number on the label! Thus, 30 added to 25 is 55, and 55 multiplied by 55 is 3025. Curious, is it not? Now, the puzzle is to find another number, composed of four figures, all different, which may be divided in the middle and produce the same result.

30. CIRCLING THE SQUARES.



The puzzle is to place a different number in each of the ten squares so that the sum of the squares of any two adjacent numbers shall be equal to the sum of the squares of the two numbers diametrically opposite to them. The four numbers placed, as examples, must stand as they are. The square of 16 is 256, and the square of 2 is 4. Add these together, and the result is 260. Also—the square of 14 is 196, and the square of 8 is 64. These together also make 260. Now, in precisely the same way, B and C should be equal to G and H (the sum will not necessarily be 260), A and K to F and E, H and I to C and D, and so on, with any two adjoining squares in the circle.

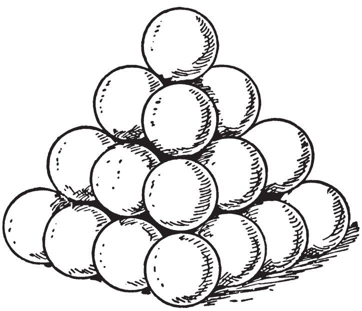
All you have to do is to fill in the remaining six numbers. Fractions are not allowed, and I shall show that no number need contain more than two figures.

31. THE FARMER AND HIS SHEEP.

Farmer Longmore had a curious aptitude for arithmetic, and was known in his district as the 'mathematical farmer.' The new vicar was not aware of this fact when, meeting his worthy parishioner one day in the lane, he asked him in the course of a short conversation, 'Now, how many sheep have you altogether?' He was therefore rather surprised at Longmore's answer, which was as follows: 'You can divide my sheep into two different parts, so that the difference between the two numbers is the same as the difference between their squares. Maybe, Mr. Parson, you will like to work out the little sum for yourself.'

Can the reader say just how many sheep the farmer had? Supposing he had possessed only twenty sheep, and he divided them into the two parts 12 and 8. Now, the difference between their squares, 144 and 64, is 80. So that will not do, for 4 and 80 are certainly not the same. If you can find numbers that work out correctly, you will know exactly how many sheep Farmer Longmore owned.

32. THE ARTILLERYMEN'S DILEMMA.



'All cannon-balls are to be piled in square pyramids,' was the order issued to the regiment. This was done. Then came the further order, 'All pyramids are to contain a square number of balls.' Whereupon the trouble arose. 'It can't be done,' said the major. 'Look at this pyramid, for example; there are sixteen balls at the base, then nine, then four, then one at the top, making thirty balls in all. But there must be six more balls, or five fewer, to make a square number.' 'It must be done,' insisted the general. 'All you have to do is to put the right number of balls in your pyramids.' 'I've got it!' said a lieutenant, the mathematical genius of the regiment. 'Lay the balls out singly.' 'Bosh!' exclaimed the general. 'You can't pile one ball into a pyramid!' Is it really possible to obey both orders?

GEOMETRICAL PROBLEMS

'God geometrizes continually.'

PLATO.

'There is no study,' said Augustus de Morgan, 'which presents so simple a beginning as that of geometry; there is none in which difficulties grow more rapidly as we proceed.' This will be found when the reader comes to consider the following puzzles, though they are not arranged in strict order of difficulty. And the fact that they have interested and given pleasure to man for untold ages is no doubt due in some measure to the appeal they make to the eye as well as to the brain. Sometimes an algebraical formula or theorem seems to give pleasure to the mathematician's eye, but it is probably only an intellectual pleasure. But there can be no doubt that in the case of certain geometrical problems, notably dissection or superposition puzzles, the æsthetic faculty in man contributes to the delight. Law and order in Nature are always pleasing to contemplate, but when they come under the very eye they seem to make a specially strong appeal. Even the person with no geometrical knowledge whatever is induced after the inspection of such things to exclaim, 'How very pretty!' In fact, I have known more than one person led on to a study of geometry by the fascination of cutting-out puzzles. I have, therefore, thought it well to keep these dissection puzzles distinct from the geometrical problems on more general lines.

DISSECTION PUZZLES

'Take him and cut him out in little stars.'
Romeo and Juliet, iii. 2.

Puzzles have infinite variety, but perhaps there is no class more ancient than dissection, cutting-out, or superposition puzzles. They were certainly known to the Chinese several thousand years before the Christian era. And they are just as fascinating to-day as they can have been at any period of their history. It is supposed by those who have investigated the matter that the ancient Chinese philosophers used these puzzles as a sort of kindergarten method of imparting the principles of geometry. Whether this was so or not, it is certain that all good dissection puzzles (for the nursery type of jig-saw puzzle, which merely consists in cutting up a picture into pieces to be put together again, is not worthy of serious consideration) are really based on geometrical laws. This statement need not, however, frighten off the novice, for it means little more than this, that geometry will give us the 'reason why,' if we are interested in knowing it, though the solutions may often be discovered by any intelligent person after the exercise of patience, ingenuity, and common sagacity.

If we want to cut one plane figure into parts that by readjustment will form another figure, the first thing is to find a way of doing it at all, and then to discover how to do it in the fewest possible pieces. Often a dissection problem is quite easy apart from this limitation of pieces. At the time of the publication in the *Weekly Dispatch*, in 1902, of a method of cutting an equilateral triangle into four parts that will form a square (see No. 26, 'Canterbury Puzzles'), no geometrician would have had any difficulty in doing what is required in five pieces: the whole point of the discovery lay in performing the little feat in four pieces only.

Mere approximations in the case of these problems are valueless; the solution must be geometrically exact, or it is not a solution at all. Fallacies are cropping up now and again, and I shall have occasion to refer to one or two of these. They are interesting merely as fallacies. But I want to say something on two little points that are always arising in cutting-out puzzles—the questions of 'hanging by a thread' and 'turning over.' These points can best be illustrated by a puzzle that is frequently to be found in the old books, but invariably with a false solution. The puzzle is to cut the figure shown in Fig. 1 into three pieces that will fit together and form a half-square triangle. The answer that is invariably given is that shown in Figs. 1 and 2. Now, it is claimed that the four pieces marked C are really only one piece, because they may be so cut that they are left 'hanging together by a mere thread.' But no serious puzzle lover will ever admit this. If the cut is made so as to leave the four pieces joined in one, then it cannot result in a perfectly exact solution. If, on the other hand, the solution is to be exact, then there will be four pieces—or six pieces in all. It is, therefore, not a solution in three pieces.

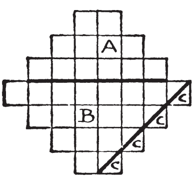


Fig. 1

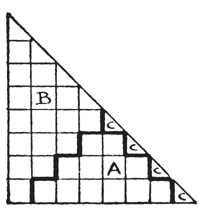


Fig. 2

If, however, the reader will look at the solution in Figs. 3 and 4, he will see that no such fault can be found with it. There is no question whatever that there are three pieces, and the solution is in this respect quite satisfactory. But another question arises. It will be found on inspection that the piece marked F, in Fig. 3, is turned over in Fig. 4–that is to say, a different side has necessarily to be presented. If the puzzle were merely to be cut out of cardboard or wood, there might be no objection to this reversal, but it is quite possible that the material would not admit of being reversed. There might be a pattern, a polish, a difference of texture, that prevents it. But it is generally understood that in dissection puzzles you are allowed to turn pieces over unless it is distinctly stated that you may not do so. And very often a puzzle is greatly improved by the added condition, 'no piece may be turned over.' I have often made puzzles, too, in which the diagram has a small repeated pattern, and the pieces have then so to be cut that not only is there no turning over, but the pattern has to be matched, which cannot be done if the pieces are turned round, even with the proper side uppermost.

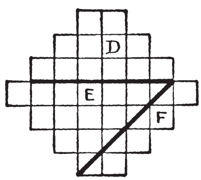


Fig. 3

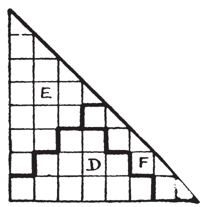
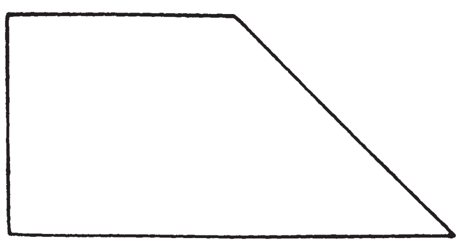


Fig. 4

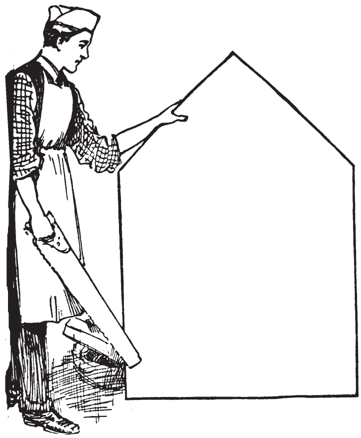
We will now consider a small miscellaneous selection of cutting-out puzzles, varying in degrees of difficulty.

33. AN EASY DISSECTION PUZZLE.



First, cut out a piece of paper or cardboard of the shape shown in the illustration. It will be seen at once that the proportions are simply those of a square attached to half of another similar square, divided diagonally. The puzzle is to cut it into four pieces all of precisely the same size and shape.

34. THE JOINER'S PROBLEM.

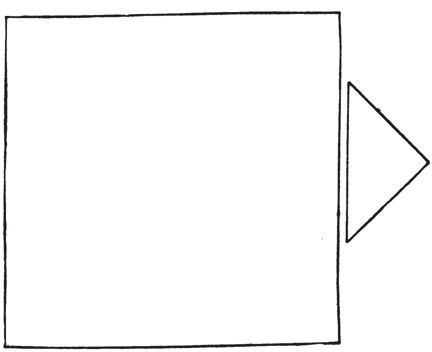


I have often had occasion to remark on the practical utility of puzzles, arising out of an application to the ordinary affairs of life of the little tricks and 'wrinkles' that we learn while solving recreation problems.

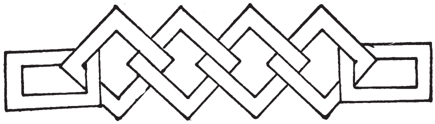
The joiner, in the illustration, wants to cut the piece of wood into as few pieces as possible to form a square table-top, without any waste of material. How should he go to work? How many pieces would you require?

35. ANOTHER JOINER'S PROBLEM.

A joiner had two pieces of wood of the shapes and relative proportions shown in the diagram. He wished to cut them into as few pieces as possible so that they could be fitted together, without waste, to form a perfectly square table-top. How should he have done it? There is no necessity to give measurements, for if the smaller piece (which is half a square) be made a little too large or a little too small it will not affect the method of solution.

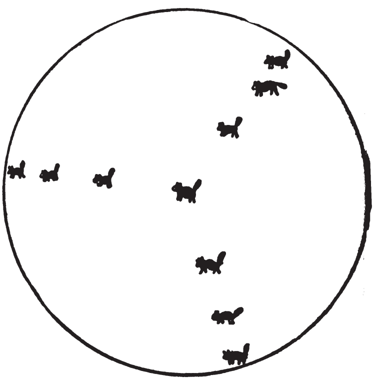


36. THE CARDBOARD CHAIN.



Can you cut this chain out of a piece of cardboard without any join whatever? Every link is solid; without its having been split and afterwards joined at any place. It is an interesting old puzzle that I learnt as a child, but I have no knowledge as to its inventor.

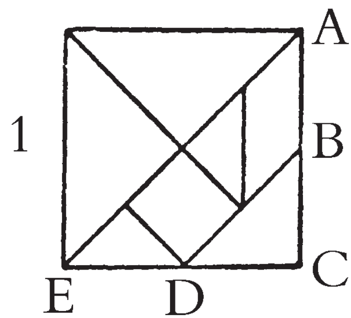
37. THE WIZARD'S CATS.



A wizard placed ten cats inside a magic circle as shown in our illustration, and hypnotized them so that they should remain stationary during his pleasure. He then proposed to draw three circles inside the large one, so that no cat could approach another cat without crossing a magic circle. Try to draw the three circles so that every cat has its own enclosure and cannot reach another cat without crossing a line.

38. A TANGRAM PARADOX.

Many pastimes of great antiquity, such as chess, have so developed and changed down the centuries that their original inventors would scarcely recognize them. This is not the case with Tangrams, a recreation that appears to be at least four thousand years old, that has apparently never been dormant, and that has not been altered or 'improved upon' since the legendary Tan first cut out the seven pieces shown in Diagram I. If you mark the point B, midway between A and C, on one side of a square of any size, and D, midway between C and E, on an adjoining side, the direction of the cuts is too obvious to need further explanation. Every design in this article is built up from the seven pieces of blackened cardboard. It will at once be understood that the possible combinations are infinite.

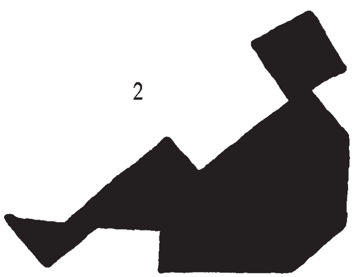


The late Mr. Sam Loyd, of New York, who published a small book of very ingenious designs, possessed the manuscripts of the late Mr. Challenor, who made a long and close study of Tangrams. This gentleman, it is said, records that there were originally seven books of Tangrams, compiled in China two thousand years before the Christian era. These books are so rare that, after forty years' residence in the country, he only succeeded in seeing perfect copies of the first and seventh volumes with fragments of the second. Portions of one of the books, printed in gold leaf upon parchment, were found in Peking by an English soldier and sold for three hundred pounds.

A few years ago a little book came into my possession, from the library of the late Lewis Carroll, entitled *The Fashionable Chinese Puzzle*. It contains three hundred and twenty-three Tangram designs, mostly nondescript geometrical figures, to be constructed from the seven pieces. It was 'Published by J. and E. Wallis, 42 Skinner Street, and J. Wallis, Jun., Marine Library, Sidmouth' (South Devon). There is no date, but the following note fixes the time of publication pretty closely: 'This ingenious contrivance has for some time past been the favourite amusement of the ex-Emperor Napoleon, who, being now in a debilitated state and living very retired, passes many hours a day in thus exercising his patience and ingenuity.' The reader will find, as did the great exile, that much amusement, not wholly uninstructive, may be derived from forming the designs of others. He will find many of the illustrations to this article quite easy to build up, and some rather difficult. Every picture may thus be regarded as a puzzle.

But it is another pastime altogether to create new and original designs of a pictorial character, and it is surprising what extraordinary scope the Tangrams afford for producing pictures of real life—angular and often grotesque, it is true, but full of character. I give an example of a recumbent figure (2) that is particularly graceful, and only needs some slight reduction of its angularities to produce an entirely satisfactory outline.

As I have referred to the author of *Alice in Wonderland*, I give also my designs of the March Hare (3) and the Hatter (4). I also give an attempt at Napoleon (5), and a very excellent Red Indian with his Squaw by Mr. Loyd (6 and 7). A large number of other designs will be found in an article by me in *The Strand Magazine* for November, 1908.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On the appearance of this magazine article, the late Sir James Murray, the eminent philologist, tried, with that amazing industry that characterized all his work, to trace the word 'tangram' to its source. At length he wrote as follows:—'One

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of my sons is a professor in the Anglo-Chinese college at Tientsin. Through him, his colleagues, and his students, I was able to make inquiries as to the alleged Tan among Chinese scholars. Our Chinese professor here (Oxford) also took an interest in the matter and obtained information from the secretary of the Chinese Legation in London, who is a very eminent representative of the Chinese literati.'

'The result has been to show that the man Tan, the god Tan, and the 'Book of Tan' are entirely unknown to Chinese literature, history, or tradition. By most of the learned men the name, or allegation of the existence, of these had never been heard of. The puzzle is, of course, well known. It is called in Chinese *ch'i ch'iao t'u*; literally, 'seven-ingenious-plan' or 'ingenious-puzzle figure of seven pieces.' No name approaching 'tangram,' or even 'tan,' occurs in Chinese, and the only suggestions for the latter were the Chinese t'an, 'to extend'; or t'ang, Cantonese dialect for 'Chinese.' It was suggested that probably some American or Englishman who knew a little Chinese or Cantonese, wanting a name for the puzzle, might concoct one out of one of these words and the European ending 'gram.' I should say the name 'tangram' was probably invented by an American some little time before 1864 and after 1847, but I cannot find it in print before the 1864 edition of Webster. I have therefore had to deal very shortly with the word in the dictionary, telling what it is applied to and what conjectures or guesses have been made at the name, and giving a few quotations, one from your own article, which has enabled me to make more of the subject than I could otherwise have done.'

6

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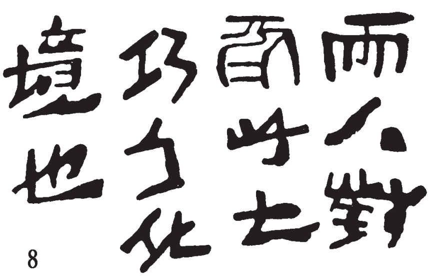
Several correspondents have informed me that they possess, or had possessed, specimens of the old Chinese books. An American gentleman writes to me as follows:—'I have in my possession a book made of tissue paper, printed in black (with a Chinese inscription on the front page), containing over three hundred designs, which belongs to the box of 'tangrams,' which I also own. The blocks are seven in number, made of mother-of-pearl, highly polished and finely engraved on either side. These are contained in a rosewood box 2 $\frac{1}{8}$ in. square. My great uncle was one of the first missionaries to visit China. This box and book, along with quite a collection of other relics, were sent to my grandfather and descended to myself.'

My correspondent kindly supplied me with rubbings of the Tangrams, from which it is clear that they are cut in the exact proportions that I have indicated. I reproduce the Chinese inscription (8) for this reason.



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By using several sets of Tangrams at the same time we may construct more ambitious pictures. I was advised by a friend

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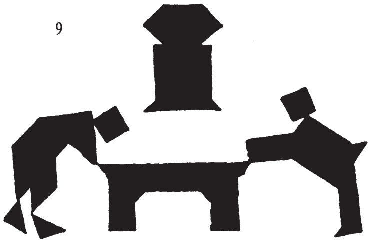
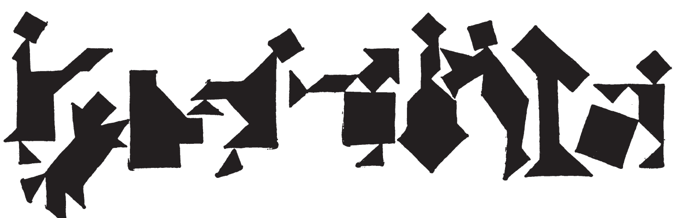
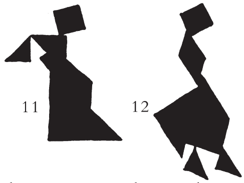


not to send my picture, 'A Game of Billiards' (9), to the Academy. He assured me that it would not be accepted because the 'judges are so hide-bound by convention.' Perhaps he was right, and it will be more appreciated by Post-impressionists and Cubists. The players are considering a very delicate stroke at the top of the table. Of course, the two men, the table, and the clock are formed from four sets of Tangrams. My second picture is named 'The Orchestra' (10), and it was designed for the decoration of a large hall of music. Here we have the conductor, the pianist, the fat little cornet-player, the left-handed player of the double-bass, whose attitude is life-like, though he does stand at an unusual distance from his instrument, and the drummer-boy, with his imposing music-stand. The dog at the back of the pianoforte is not howling: he is an appreciative listener.



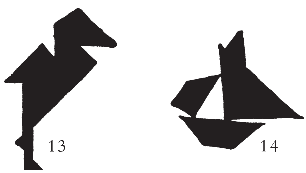
10

One remarkable thing about these Tangram pictures is that they suggest to the imagination such a lot that is not really there. Who, for example, can look for a few minutes at Lady Belinda (11) and the Dutch girl (12) without soon feeling the haughty expression in the one case and the arch look in the other? Then



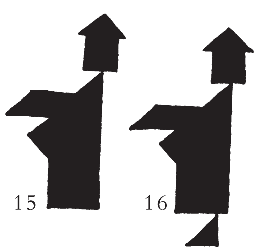
11 12

look again at the stork (13), and see how it is suggested to the mind that the leg is actually much more slender than any one of the pieces employed. It is really an optical illusion. Again, notice in the case of the yacht (14) how, by leaving that little angular point at the top, a complete mast is suggested. If you place your Tangrams together on white paper so that they do not quite touch one another, in some cases the effect is improved by the white lines; in other cases it is almost destroyed.



13 14

Finally, I give an example from the many curious paradoxes that one happens upon in manipulating Tangrams. I show designs of two dignified individuals (15 and 16) who appear to be exactly alike, except for the fact that one has a foot and the other has not. Now, both of these figures are made from the same seven Tangrams. Where does the second man get his foot from?



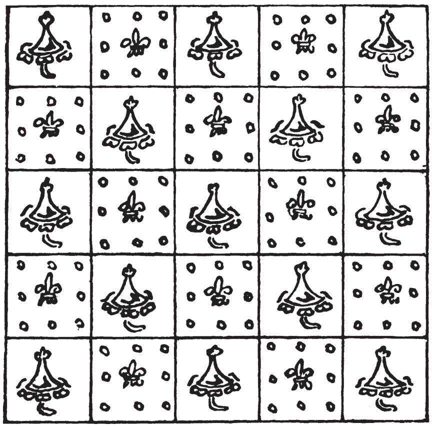
15 16

PATCHWORK PUZZLES

'Of shreds and patches.'

—*Hamlet*, iii. 4.

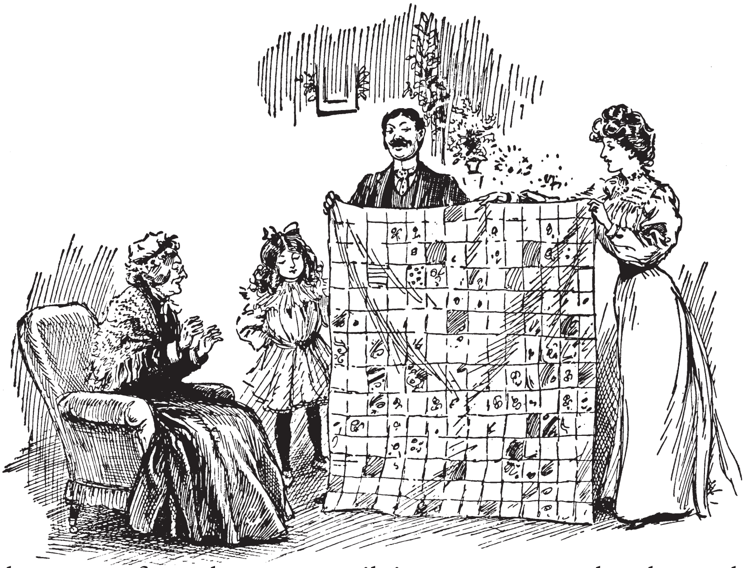
39. THE CUSHION COVERS.



The above represents a square of brocade. A lady wishes to cut it in four pieces so that two pieces will form one perfectly square cushion top, and the remaining two pieces another square cushion top. How is she to do it? Of course, she can only cut along the lines that divide the twenty-five squares, and the pattern must 'match' properly without any irregularity whatever in the design of the material. There is only one way of doing it. Can you find it?

40. MRS. SMILEY'S CHRISTMAS PRESENT.

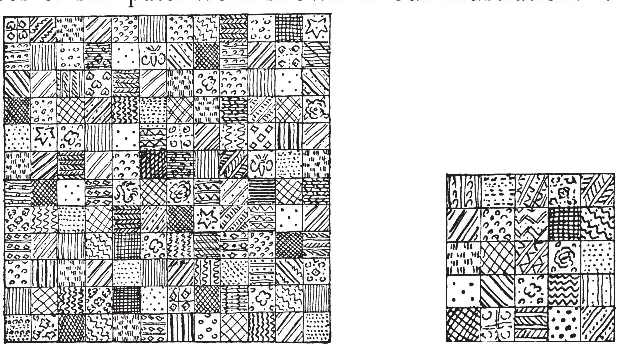
Mrs. Smiley's expression of pleasure was sincere when her six granddaughters sent to her, as a Christmas present, a very pretty patchwork quilt, which they had made with their own hands. It was constructed of square pieces of silk material, all of one size, and as they made a large quilt with fourteen of these little squares on each side, it is obvious that just 196 pieces had been stitched into it. Now, the six granddaughters each contributed a part of the work in the form of a perfect square (all six portions being different in size), but in order to join



them up to form the square quilt it was necessary that the work of one girl should be unpicked into three separate pieces. Can you show how the joins might have been made? Of course, no portion can be turned over.

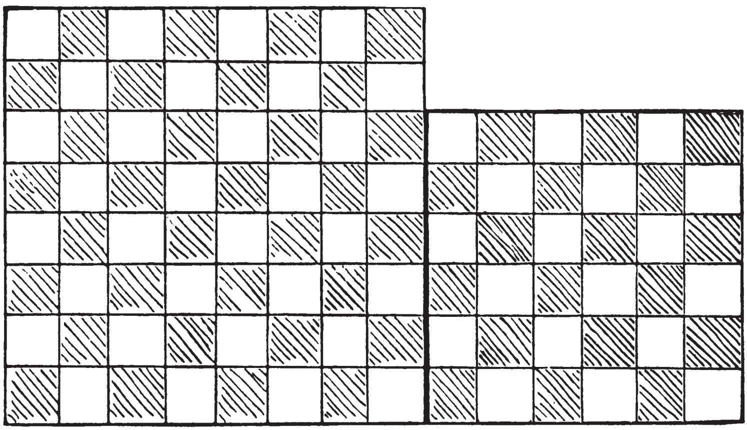
41. ANOTHER PATCHWORK PUZZLE.

A lady was presented, by two of her girl friends, with the pretty pieces of silk patchwork shown in our illustration. It will be



seen that both pieces are made up of squares all of the same size—one 12 × 12 and the other 5 × 5. She proposes to join them together and make one square patchwork quilt, 13 × 13, but, of course, she will not cut any of the material—merely cut the stitches where necessary and join together again. What perplexes her is this. A friend assures her that there need be no more than four pieces in all to join up for the new quilt. Could you show her how this little needlework puzzle is to be solved in so few pieces?

42. LINOLEUM CUTTING.



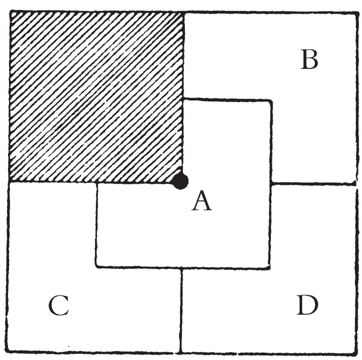
The diagram herewith represents two separate pieces of linoleum. The chequered pattern is not repeated at the back, so that the pieces cannot be turned over. The puzzle is to cut the two squares into four pieces so that they shall fit together and form one perfect square 10 × 10, so that the pattern shall properly match, and so that the larger piece shall have as small a portion as possible cut from it.

VARIOUS GEOMETRICAL PUZZLES

'So various are the tastes of men.'
MARK AKENSIDE.

43. THE FOUR SONS.

A man possessed a square-shaped estate. He bequeathed to his widow the quarter of it that is shaded off. The remainder was to be divided equitably amongst his four sons, so that each should receive land of exactly the same area and exactly similar in shape. We are shown how this was done. In the centre of the estate was a well, indicated by the dark spot, and Benjamin, Charles, and David complained that the division was not 'equitable,' since Alfred had access to this well, while they could not reach it without trespassing on somebody else's land. The puzzle is to show how the estate is to be apportioned so that each son shall have land of the same shape and area, and each have access to the well without going off his own land.



44. THE THREE RAILWAY STATIONS.

As I sat in a railway carriage I noticed at the other end of the compartment a worthy squire, whom I knew by sight, engaged in conversation with another passenger, who was evidently a friend of his.

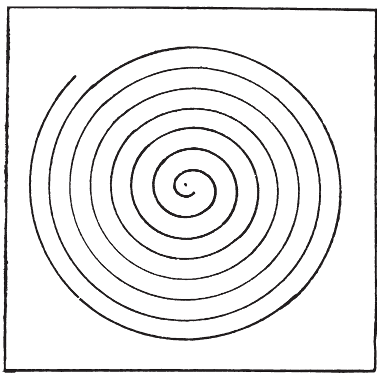
'How far have you to drive to your place from the railway station?' asked the stranger.

'Well,' replied the squire, 'if I get out at Appleford, it is just the same distance as if I go to Bridgefield, another fifteen miles farther on; and if I changed at Appleford and went thirteen miles from there to Carterton, it would still be the same distance. You see, I am equidistant from the three stations, so I get a good choice of trains.'

Now I happened to know that Bridgefield is just fourteen miles from Carterton, so I amused myself in working out the exact distance that the squire had to drive home whichever station he got out at. What was the distance?

45. DRAWING A SPIRAL.

If you hold the page horizontally and give it a quick rotary motion while looking at the centre of the spiral, it will appear to revolve. Perhaps a good many readers are

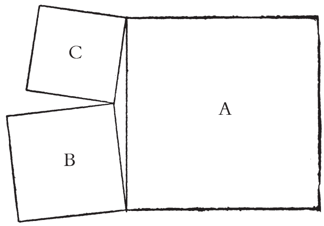


acquainted with this little optical illusion. But the puzzle is to show how I was able to draw this spiral with so much exactitude without using anything but a pair of compasses and the sheet of paper on which the diagram was made. How would you proceed in such circumstances?

46. THE YORKSHIRE ESTATES.

I was on a visit to one of the large towns of Yorkshire. While walking to the railway station on the day of my departure a man thrust a hand-bill upon me, and I took this into the railway carriage and read it at my leisure. It informed me that three Yorkshire neighbouring estates were to be offered for sale. Each estate was square in shape, and they joined one another at their corners, just as shown in the diagram. Estate

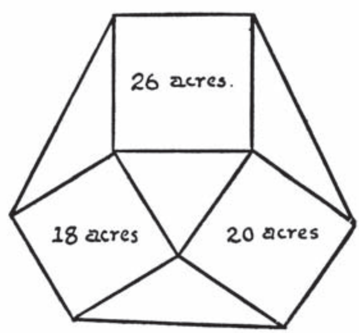
A contains exactly 370 acres, B contains 116 acres, and C 74 acres.



Now, the little triangular bit of land enclosed by the three square estates was not offered for sale, and, for no reason in particular, I became curious as to the area of that piece. How many acres did it contain?

47. FARMER WURZEL'S ESTATE.

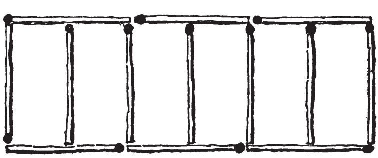
I will now present another land problem. The demonstration of the answer that I shall give will, I think, be found both interesting and easy of comprehension.



Farmer Wurzel owned the three square fields shown in the annexed plan, containing respectively 18, 20, and 26 acres. In order to get a ring-fence round his property he bought the four intervening triangular fields. The puzzle is to discover what was then the whole area of his estate.

48. THE SIX SHEEP-PENS.

Here is a new little puzzle with matches. It will be seen in the illustration that thirteen matches, representing a farmer's hurdles, have been so placed that they enclose six sheep-pens all of the same size. Now, one of these hurdles was stolen, and the farmer wanted still to



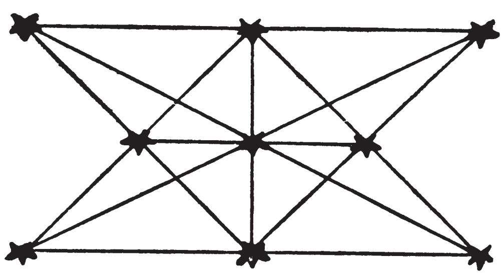
enclose six pens of equal size with the remaining twelve. How was he to do it? All the twelve matches must be fairly used, and there must be no duplicated matches or loose ends.

POINTS AND LINES PROBLEMS

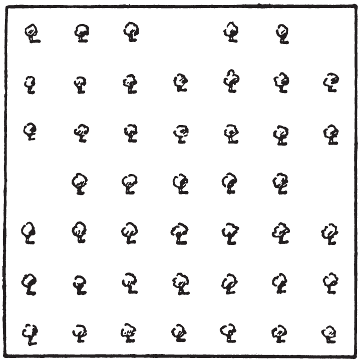
'Line upon line, line upon line; here a little and there a little.'
—*Isa*. xxviii. 10.

What are known as 'Points and Lines' puzzles are found very interesting by many people. The most familiar example, here given, to plant nine trees so that they shall form ten straight rows with three trees in every row, is attributed to Sir Isaac Newton, but the earliest collection of such puzzles is, I believe, in a rare little book that I possess—published in 1821—*Rational Amusement for Winter Evenings*, by John Jackson. The author gives ten examples of 'Trees planted in Rows.'

These tree-planting puzzles have always been a matter of great perplexity. They are real 'puzzles,' in the truest sense of the word, because nobody has yet succeeded in finding a direct and certain way of solving them. They demand the exercise of sagacity, ingenuity, and patience, and what we call 'luck' is also sometimes of service. Perhaps some day a genius will discover the key to the whole mystery. Remember that the trees must be regarded as mere points, for if we were allowed to make our trees big enough we might easily 'fudge' our diagrams and get in a few extra straight rows that were more apparent than real.



49. A PLANTATION PUZZLE.

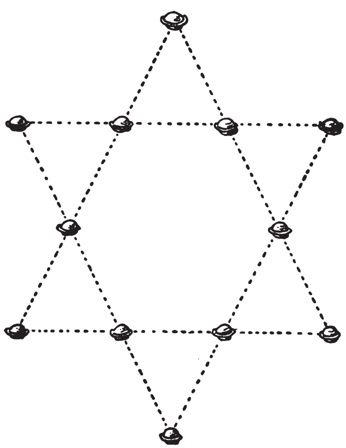


A man had a square plantation of forty-nine trees, but, as will be seen by the omissions in the illustration, four trees were blown down and removed. He now wants to cut down all the remainder except ten trees, which are to be so left that they shall form five straight rows with four trees in every row. Which are the ten trees that he must leave?

50. THE TWENTY-ONE TREES.

A gentleman wished to plant twenty-one trees in his park so that they should form twelve straight rows with five trees in every row. Could you have supplied him with a pretty symmetrical arrangement that would satisfy these conditions?

51. THE TWELVE MINCE-PIES.



It will be seen in our illustration how twelve mince-pies may be placed on the table so as to form six straight rows with four pies in every row. The puzzle is to remove only four of them to new positions so that there shall be seven straight rows with four in every row. Which four would you remove, and where would you replace them?

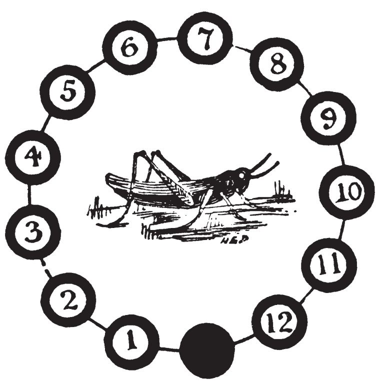
MOVING COUNTER PROBLEMS

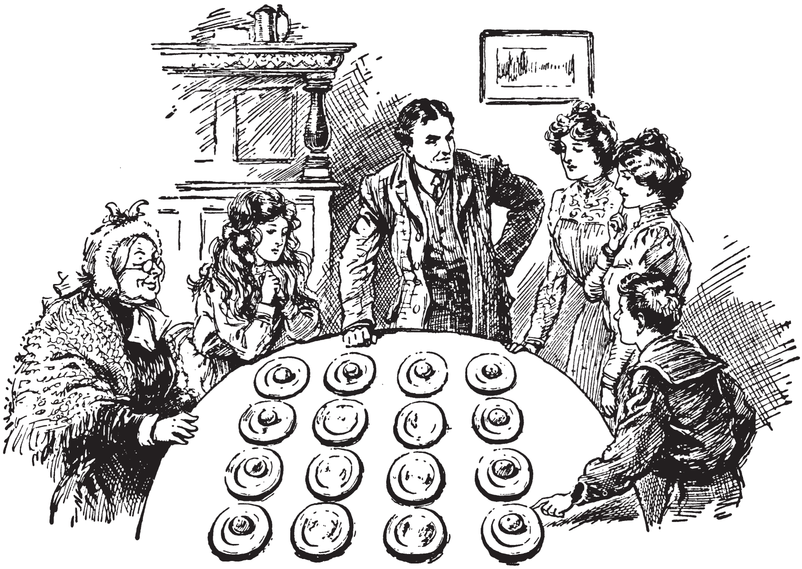
'I cannot do't without counters.'

Winter's Tale, iv. 3.

52. THE GRASSHOPPER PUZZLE.

It has been suggested that this puzzle was a great favourite among the young apprentices of the City of London in the sixteenth and seventeenth centuries. Readers will have noticed the curious brass grasshopper on the Royal Exchange. This long-lived creature escaped the fires of 1666 and 1838. The grasshopper, after his kind, was the crest of Sir Thomas Gresham, merchant grocer, who died in 1579, and from this cause it has been used as a sign by grocers in general. Unfortunately for the legend as to its origin, the puzzle was only produced by myself so late as the year 1900. On twelve of the thirteen black discs are placed numbered counters or grasshoppers. The puzzle is to reverse their order, so that they shall read, 1, 2, 3, 4, etc., in the opposite direction, with the vacant disc left in the same position as at present. Move one at a time in any order, either to the adjoining vacant disc or by jumping over one grasshopper, like the moves in draughts. The moves or leaps may be made in either direction that is at any time possible. What are the fewest possible moves in which it can be done?

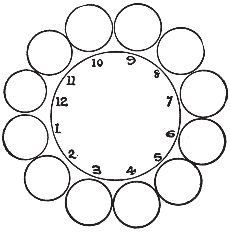




53. THE TEN APPLES.

The family represented in the illustration are amusing themselves with this little puzzle, which is not very difficult but quite interesting. They have, it will be seen, placed sixteen plates on the table in the form of a square, and put an apple in each of ten plates. They want to find a way of removing all the apples except one by jumping over one at a time to the next vacant square, as in draughts; or, better, as in solitaire, for you are not allowed to make any diagonal moves—only moves parallel to the sides of the square. It is obvious that as the apples stand no move can be made, but you are permitted to transfer any single apple you like to a vacant plate before starting. Then the moves must be all leaps, taking off the apples leaped over.

54. THE TWELVE PENNIES.



Here is a pretty little puzzle that only requires twelve pennies or counters. Arrange them in a circle, as shown in the illustration. Now take up one penny at a time and, passing it over two pennies, place it on the third penny. Then take up another single penny and do the same thing, and so on, until, in six such moves, you have the coins in six pairs in the positions 1, 2, 3, 4, 5, 6. You can move in either direction round the circle at every play, and it does not matter whether the two jumped over are separate or a pair. This is quite easy if you use just a little thought.

55. THE ECCENTRIC CHEESEMONGER.

The cheesemonger depicted in the illustration is an inveterate puzzle lover. One of his favourite puzzles is the piling of cheeses in his warehouse, an amusement that he finds good exercise for the body as well as for the mind. He places sixteen cheeses on the floor in a straight row and then makes them into four piles, with four cheeses in every pile, by always passing a cheese over four others. If you use sixteen counters and number them in order from 1 to 16, then you may place 1 on 6, 11 on 1, 7 on 4, and so on, until there are four in every pile. It will be seen that it does not matter whether the four passed over are standing alone or piled; they count just the same, and you can always carry a cheese in either direction. There are a great many different ways of doing it in twelve moves, so it makes a good game of 'patience' to try to solve it so that the four piles shall be left in different stipulated places. For example, try to leave the piles at the extreme ends of the row, on Nos. 1, 2, 15 and 16; this is quite easy. Then try to leave three piles together, on Nos. 13, 14, and 15. Then again play so that they shall be left on Nos. 3, 5, 12, and 14.

56. THE HAT PUZZLE.

Ten hats were hung on pegs as shown in the illustration—five silk hats and five felt 'bowlers,' alternately silk and felt. The two pegs at the end of the row were empty.

The puzzle is to remove two contiguous hats to the vacant pegs, then two other adjoining hats to the pegs now unoccupied, and so on until five pairs have been moved and the hats again hang in an unbroken row, but with all the silk ones together and all the felt hats together.

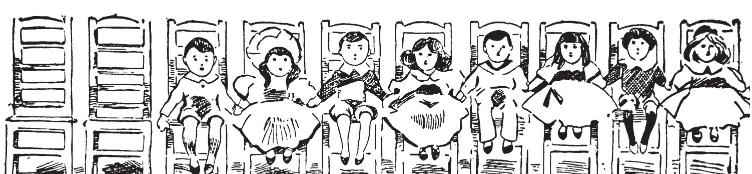
Remember, the two hats removed must always be contiguous ones, and you must take one in each hand and place them on their new pegs without reversing their relative position. You are not allowed to cross your hands, nor to hang up one at a time.

Can you solve this old puzzle, which I give as introductory to the next? Try it with counters of two colours or with coins, and remember that the two empty pegs must be left at one end of the row.

57. BOYS AND GIRLS.

If you mark off ten divisions on a sheet of paper to represent the chairs, and use eight numbered counters for the children, you will have a fascinating pastime. Let the odd numbers represent boys and even numbers girls, or you can use counters of two colours, or coins.

The puzzle is to remove two children who are occupying adjoining chairs and place them in two empty chairs, making them first change sides; then remove a second pair of children from adjoining chairs and place them in the two now vacant, making them change sides; and so on, until all the boys are together and all the girls together, with the two vacant chairs at one end as at present. To solve the puzzle you must do this in five moves. The two children must always be taken from chairs that are next to one another; and remember the important point of making the two children change sides, as this latter is the distinctive feature of the puzzle. By 'change sides' I simply mean that if, for example, you first move 1 and 2 to the vacant chairs, then the first (the outside) chair will be occupied by 2 and the second one by 1.



UNICURSAL AND ROUTE PROBLEMS

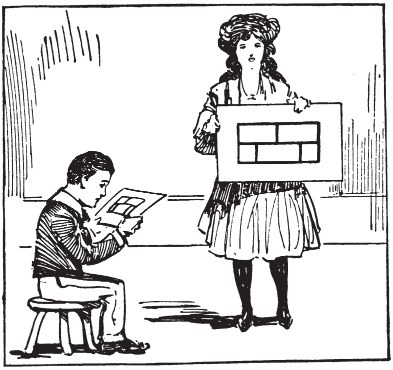
'I see them on their winding way.'
REGINALD HEBER.

It is reasonable to suppose that from the earliest ages one man has asked another such questions as these: 'Which is the nearest way home?' 'Which is the easiest or pleasantest way?' 'How can we find a way that will enable us to dodge the mastodon and the plesiosaurus?' 'How can we get there without ever crossing the track of the enemy?' All these are elementary route problems, and they can be turned into good puzzles by the introduction of some conditions that complicate matters. A variety of such complications will be found in the following examples. I have also included some enumerations of more or less difficulty. These afford excellent practice for the reasoning faculties, and enable one to generalize in the case of symmetrical forms in a manner that is most instructive.

58. A JUVENILE PUZZLE.

For years I have been perpetually consulted by my juvenile friends about this little puzzle. Most children seem to know it, and yet, curiously enough, they are invariably unacquainted with the answer. The question they always ask is, 'Do, please, tell me whether it is really possible.' I believe Houdin the conjurer used to be very fond of giving it to his child friends, but I cannot say whether he invented the little puzzle or not. No doubt a large number of my readers will be glad to have the mystery of the solution cleared up, so I make no apology for introducing this old 'teaser.'

The puzzle is to draw with three strokes of the pencil the diagram that the little girl is exhibiting in the illustration. Of

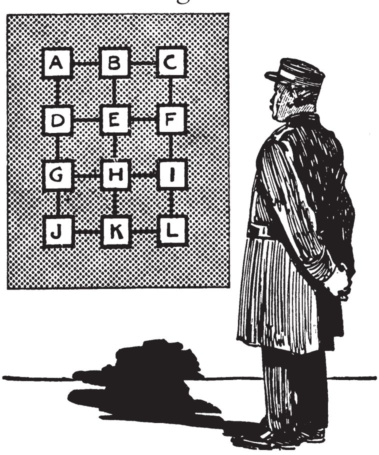


course, you must not remove your pencil from the paper during a stroke or go over the same line a second time. You will find that you can get in a good deal of the figure with one continuous stroke, but it will always appear as if four strokes are necessary.

Another form of the puzzle is to draw the diagram on a slate and then rub it out in three rubs.

59. THE TUBE INSPECTOR'S PUZZLE.

The man in our illustration is in a little dilemma. He has just been appointed inspector of a certain system of tube railways, and it is his duty to inspect regularly, within a stated period, all the company's seventeen lines connecting twelve stations, as shown on the big poster plan that he is contemplating. Now he wants to arrange his route so that it shall take him over all the

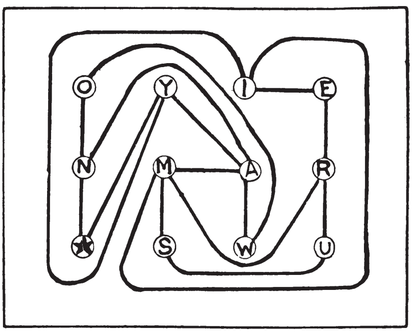


lines with as little travelling as possible. He may begin where he likes and end where he likes. What is his shortest route?

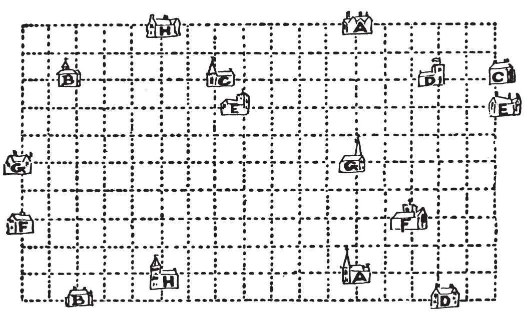
Could anything be simpler? But the reader will soon find that, however he decides to proceed, the inspector must go over some of the lines more than once. In other words, if we say that the stations are a mile apart, he will have to travel more than seventeen miles to inspect every line. There is the little difficulty. How far is he compelled to travel, and which route do you recommend?

60. THE CYCLISTS' TOUR.

Two cyclists were consulting a road map in preparation for a little tour together. The circles represent towns, and all the good roads are represented by lines. They are starting from the town with a star, and must complete their tour at E. But before arriving there they want to visit every other town once, and only once. That is the difficulty. Mr. Spicer said, 'I am certain we can find a way of doing it;' but Mr. Maggs replied, 'No way, I'm sure.' Now, which of them was correct? Take your pencil and see if you can find any way of doing it. Of course you must keep to the roads indicated.

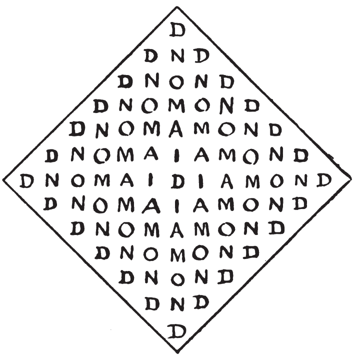


61. A PUZZLE FOR MOTORISTS.



Eight motorists drove to church one morning. Their respective houses and churches, together with the only roads available (the dotted lines), are shown. One went from his house A to his church A, another from his house B to his church B, another from C to C, and so on, but it was afterwards found that no driver ever crossed the track of another car. Take your pencil and try to trace out their various routes.

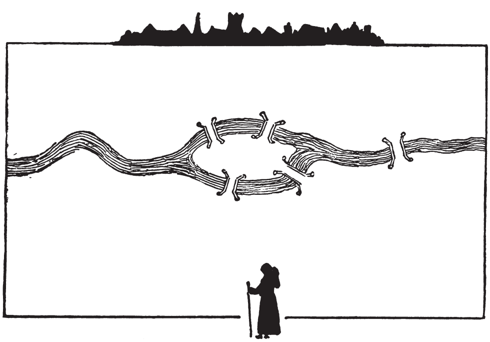
62. THE DIAMOND PUZZLE.



In how many different ways may the word DIAMOND be read in the arrangement shown? You may start wherever you like at a D and go up or down, backwards or forwards, in and out, in any direction you like, so long as you always pass from one letter to another that adjoins it. How many ways are there?

63. THE MONK AND THE BRIDGES.

In this case I give a rough plan of a river with an island and five bridges. On one side of the river is a monastery, and on the other side is seen a monk in the foreground. Now, the monk has decided that he will cross every bridge once, and only once, on his return to the monastery. This is, of course, quite easy to do, but on the way he



thought to himself, 'I wonder how many different routes there are from which I might have selected.' Could you have told him? That is the puzzle. Take your pencil and trace out a route that will take you once over all the five bridges. Then trace out a second route, then a third, and see if you can count all the variations. You will find that the difficulty is twofold: you have to avoid dropping routes on the one hand and counting the same routes more than once on the other.

COMBINATION AND GROUP PROBLEMS

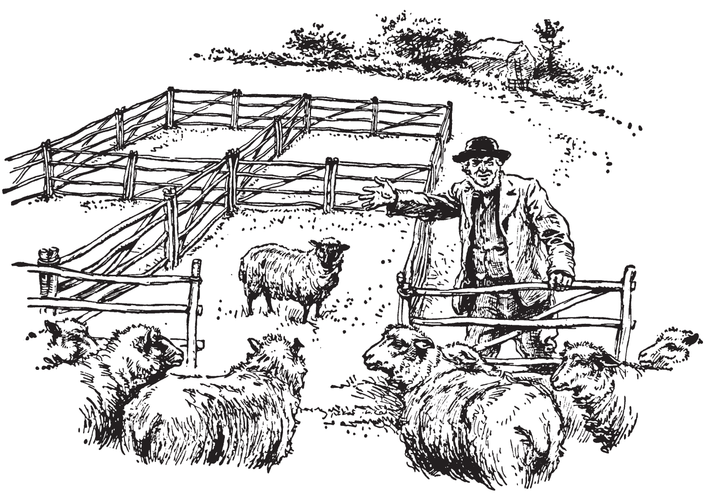
'A combination and a form indeed.'

Hamlet, iii. 4.

Various puzzles in this class might be termed problems in the 'geometry of situation,' but their solution really depends on the theory of combinations which, in its turn, is derived directly from the theory of permutations. It has seemed convenient to include here certain group puzzles and enumerations that might, perhaps, with equal reason have been placed elsewhere; but readers are again asked not to be too critical about the classification, which is very difficult and arbitrary.

64. THOSE FIFTEEN SHEEP.

A certain cyclopædia has the following curious problem, I am told: 'Place fifteen sheep in four pens so that there shall be the same



number of sheep in each pen.' No answer whatever is vouchsafed, so I thought I would investigate the matter. I saw that in dealing with apples or bricks the thing would appear to be quite impossible, since four times any number must be an even number, while fifteen is an odd number. I thought, therefore, that there must be some quality peculiar to the sheep that was not generally known. So I decided to interview some farmers on the subject. The first one pointed out that if we put one pen inside another, like the rings of a target, and placed all sheep in the smallest pen, it would be all right. But I objected to this, because you admittedly place all the sheep in one pen, not in four pens. The second man said that if I placed four sheep in each of three pens and three sheep in the last pen (that is fifteen sheep in all), and one of the ewes in the last pen had a lamb during the night, there would be the same number in each pen in the morning. This also failed to satisfy me.

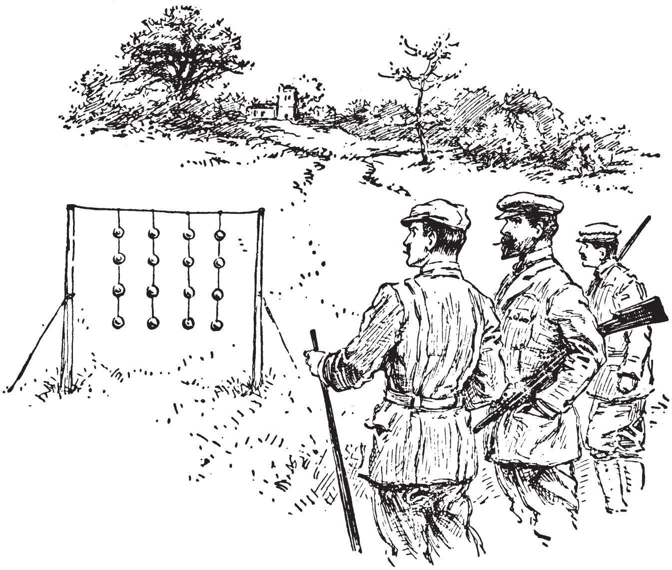
The third farmer said, 'I've got four hurdle pens down in one of my fields, and a small flock of wethers, so if you will just step down with me I will show you how it is done.' The illustration depicts my friend as he is about to demonstrate the matter to me. His lucid explanation was evidently that which was in the mind of the writer of the article in the cyclopædia. What was it? Can you place those fifteen sheep?

65. A PUZZLE FOR CARD-PLAYERS.

Twelve members of a club arranged to play bridge together on eleven evenings, but no player was ever to have the same partner more than once, or the same opponent more than twice. Can you draw up a scheme showing how they may all sit down at three tables every evening? Call the twelve players by the first twelve letters of the alphabet and try to group them.

66. THE GLASS BALLS.

A number of clever marksmen were staying at a country house, and the host, to provide a little amusement, suspended strings



of glass balls, as shown in the illustration, to be fired at. After they had all put their skill to a sufficient test, somebody asked the following question: 'What is the total number of different ways in which these sixteen balls may be broken, if we must always break the lowest ball that remains on any string?' Thus, one way would be to break all the four balls on each string in succession, taking the strings from left to right. Another would be to break all the fourth balls on the four strings first, then break the three remaining on the first string, then take the balls on the three other strings alternately from right to left, and so on. There is such a vast number of different ways (since every little variation of order makes a different way) that one is apt to be at first impressed by the great difficulty of the problem. Yet it is really quite simple when once you have hit on the proper method of attacking it. How many different ways are there?

67. FIFTEEN LETTER PUZZLE.

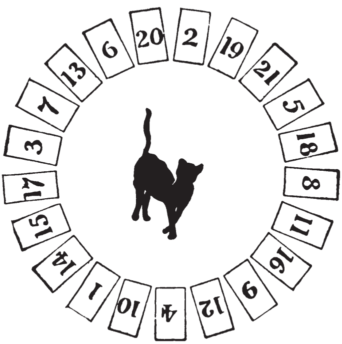
ALE	FOE	HOD	BGN
CAB	HEN	JOG	KFM
HAG	GEM	MOB	BFH
FAN	KIN	JEK	DFL
JAM	HIM	GCL	LJH
AID	JIB	FCJ	NJD
OAK	FIG	HCK	MLN
BED	OIL	MCD	BLK
ICE	CON	DGK	

The above is the solution of a puzzle I gave in *Tit-bits* in the summer of 1896. It was required to take the letters, A, B, C, D, E, F, G, H, I, J, K, L, M, N, and O, and with them form thirty-five groups of three letters so that the combinations should include the greatest number possible of common English words. No two letters may appear together in a group more than once. Thus, A and L having been together in ALE, must never be found together again; nor may A appear again in a group with E, nor L with E. These conditions will be found complied with in the above solution, and the number of words formed is twenty-one. Many persons have since tried hard to beat this number, but so far have not succeeded.

More than thirty-five combinations of the fifteen letters cannot be formed within the conditions. Theoretically, there cannot possibly be more than twenty-three words formed, because only this number of combinations is possible with a vowel or vowels in each. And as no English word can be formed from three of the given vowels (A, E, I, and O), we must reduce the number of possible words to twenty-two. This is correct theoretically, but practically that twenty-second word cannot be got in. If JEK, shown above, were a word it would be all right; but it is not, and no amount of juggling with the other letters has resulted in a better answer than the one shown. I should, say that proper nouns and abbreviations, such as Joe, Jim, Alf, Hal, Flo, Ike, etc., are disallowed.

Now, the present puzzle is a variation of the above. It is simply this: Instead of using the fifteen letters given, the reader is allowed to select any fifteen different letters of the alphabet that he may prefer. Then construct thirty-five groups in accordance with the conditions, and show as many good English words as possible.

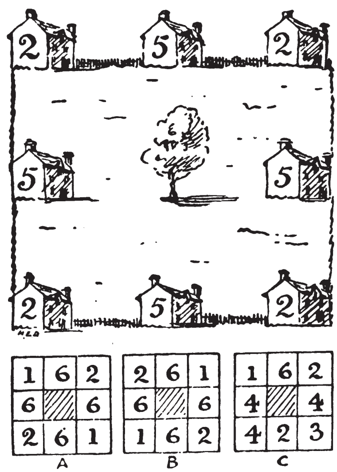
68. THE MOUSE-TRAP PUZZLE.



This is a modern version, with a difference, of an old puzzle of the same name. Number twenty-one cards, 1, 2, 3, etc., up to 21, and place them in a circle in the particular order shown in the illustration. These cards represent mice. You start from any card, calling that card 'one,' and count, 'one, two, three,' etc., in a clockwise direction, and when your count agrees with the number on the card, you have made a 'catch,' and you remove the card. Then start at the next card, calling that 'one,' and try again to make another 'catch.' And so on. Supposing you start at 18, calling that card 'one,' your first 'catch' will be 19. Remove 19 and your next 'catch' is 10. Remove 10 and your next 'catch' is 1. Remove the 1, and if you count up to 21 (you must never go beyond), you cannot make another 'catch.' Now, the ideal is to 'catch' all the twenty-one mice, but this is not here possible, and if it were it would merely require twenty-one different trials, at the most, to succeed. But the reader may make any two cards change places before he begins. Thus, you can change the 6 with the 2, or the 7 with the 11, or any other pair. This can be done in several ways so as to enable you to 'catch' all the twenty-one mice, if you then start at the right place. You may never pass over a 'catch'; you must always remove the card and start afresh.

69. THE EIGHT VILLAS.

In one of the outlying suburbs of London a man had a square plot of ground on which he decided to build eight villas, as shown in the illustration, with a common recreation ground in the middle. After the houses were completed, and all or some of them let, he discovered that the number of occupants in the three houses forming a side of the square was in every case nine. He did not state how the occupants were distributed, but I have shown by the numbers on the sides of the houses one way in which it might have happened. The puzzle is to discover the total number of ways in which all or any of the houses might be occupied, so that there should be nine persons on each side. In order that there may be no misunderstanding, I will explain that although B is what we call a reflection of A, these would count as two different arrangements, while C, if it is turned round, will give four arrangements; and if turned round in front of a mirror, four other arrangements. All eight must be counted.



CHESSBOARD PROBLEMS

During a heavy gale a chimney-pot was hurled through the air, and crashed upon the pavement just in front of a pedestrian. He quite calmly said, 'I have no use for it: I do not smoke.' Some readers, when they happen to see a puzzle represented on a chessboard with chess pieces, are apt to make the equally inconsequent remark, 'I have no use for it: I do not play chess.' But there is no condition in the game that you shall checkmate your opponent in two moves, in three moves, or in four moves. And the solving of them helps you but little, and that quite indirectly, in playing the game, it being well known that, as a rule, the best 'chess problemists' are indifferent players, and vice versa.

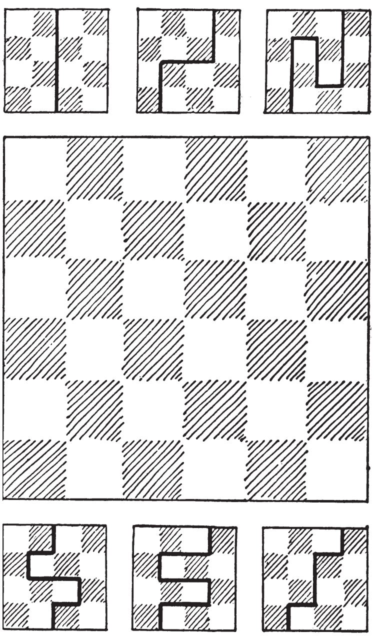
Yet the simple chequered board and the characteristic moves of the pieces lend themselves in a very remarkable manner to the devising of the most entertaining puzzles. It was with a view to securing the interest of readers who are frightened off by the mere presentation of a chessboard that so many puzzles of this class were originally published by me in various fanciful dresses. In the majority of cases the reader will not need any knowledge whatever of chess, but I have thought it best to assume throughout that he is acquainted with the terminology, the moves, and the notation of the game.

THE CHESSBOARD

'Good company's a chessboard.'
BYRON'S *Don Juan*, xiii. 89.

A chessboard is essentially a square plane divided into sixty-four smaller squares by straight lines at right angles. Originally it was not chequered (that is, made with its rows and columns alternately black and white, or of any other two colours), and this improvement was introduced merely to help the eye in actual play. The utility of the chequers is unquestionable. For example, it facilitates the operation of the bishops, enabling us to see at the merest glance that our king or pawns on black squares are not open to attack from an opponent's bishop running on the white diagonals. Yet the chequering of the board is not essential to the game of chess. Also, when we are propounding puzzles on the chessboard, it is often well to remember that additional interest may result from 'generalizing' for boards containing any number of squares, or from limiting ourselves to some particular chequered arrangement, not necessarily a square.

70. CHEQUERED BOARD DIVISIONS.

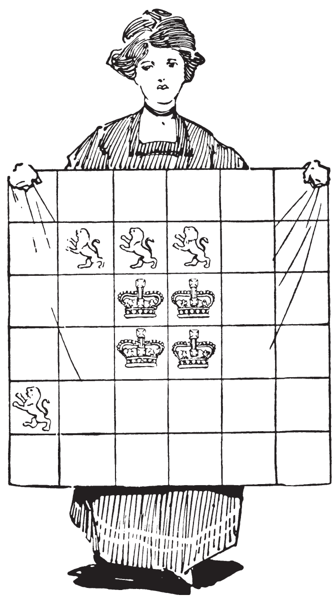


I recently asked myself the question: In how many different ways may a chessboard be divided into two parts of the same size and shape by cuts along the lines dividing the squares? The problem soon proved to be both fascinating and bristling with difficulties. I present it in a simplified form, taking a board of smaller dimensions.

It is obvious that a board of four squares can only be so divided in one way—by a straight cut down the centre—because we shall not count reversals and reflections as different. In the case of a board of sixteen squares—four by four—there are just six different ways. I have given all these in the diagram, and the reader will not find any others. Now, take the larger board of thirty-six squares, and try to discover in how many ways it may be cut into two parts of the same size and shape.

71. LIONS AND CROWNS.

The young lady in the illustration is confronted with a little cutting-out difficulty in which the reader may be glad to assist her. She wishes, for some reason that she has not communicated to me, to cut that square piece of valuable material into four parts, all of exactly the same size and shape, but it is important that every piece shall contain a lion and a crown. As she insists that the cuts can only be made along the lines dividing the squares, she is considerably perplexed to find out how it is to be done. Can you show her the way? There is only one possible method of cutting the stuff.



72. BOARDS WITH AN ODD NUMBER OF SQUARES.

We will here consider the question of those boards that contain an odd number of squares. We will suppose that the central square is first cut out, so as to leave an even number of squares for division. Now, it is obvious that a square three by three can only be divided in one way, as shown in Fig. 1. It will be seen that the pieces A and B are of the same size and shape, and that any other way of cutting would only produce the same shaped pieces, so remember that these variations are not counted as different ways. The puzzle I propose is to cut the board five by five (Fig. 2) into two pieces of the same size and shape in as many different ways as possible. I have shown in the illustration one way of doing it. How many different ways are there altogether? A piece which when turned over resembles another piece is not considered to be of a different shape.

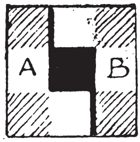


Fig. 1

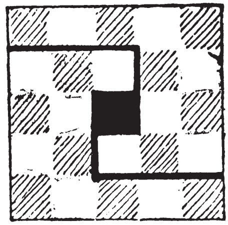
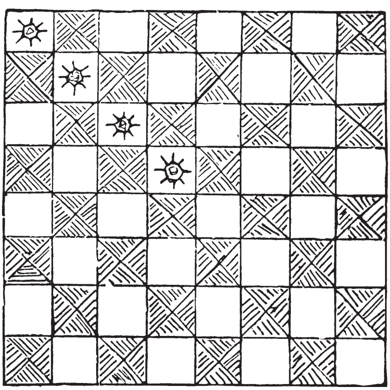


Fig. 2

73. THE GRAND LAMA'S PROBLEM.

Once upon a time there was a Grand Lama who had a chessboard made of pure gold, magnificently engraved, and, of course, of great value. Every year a tournament was held at Lhassa among the priests, and whenever any one beat the Grand Lama it was considered a great honour, and his name was inscribed on the back of the board, and a costly jewel set in the particular square on which the checkmate had been given. After this sovereign pontiff had been defeated on four occasions he died—possibly of chagrin.



Now the new Grand Lama was an inferior chess-player, and preferred other forms of innocent amusement, such as cutting off people's heads. So he discouraged chess as a degrading game, that did not improve either the mind or the morals, and abolished the tournament summarily. Then he sent for the four priests who had had the effrontery to play better than a Grand Lama, and addressed them as follows:

'Miserable and heathenish men, calling yourselves priests! Know ye not that to lay claim to a capacity to do anything better than my predecessor is a capital offence? Take that chessboard and, before day dawns upon the torture chamber, cut it into four equal parts of the same shape, each containing sixteen perfect squares, with one of the gems in each part! If in this you fail, then shall other sports be devised for your special delectation. Go!'

The four priests succeeded in their apparently hopeless task. Can you show how the board may be divided into four equal parts, each of exactly the same shape, by cuts along the lines dividing the squares, each part to contain one of the gems?

STATICAL CHESS PUZZLES

'They also serve who only stand and wait.'

MILTON.

74. THE EIGHT ROOKS.

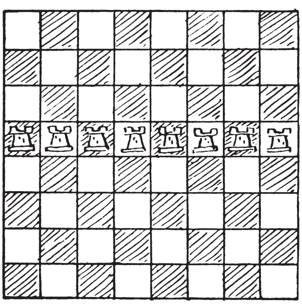


Fig. 1.

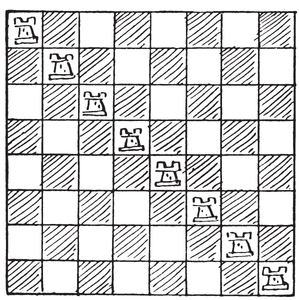
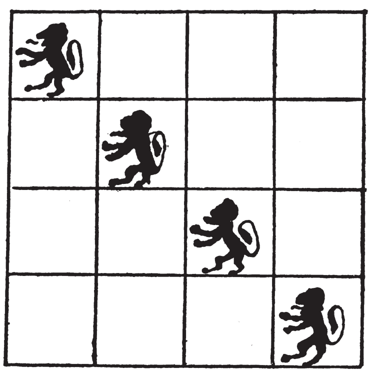


Fig. 2.

It will be seen in the first diagram that every square on the board is either occupied or attacked by a rook, and that every rook is 'guarded' (if they were alternately black and white rooks we should say 'attacked') by another rook. Placing the eight rooks on any row or file obviously will have the same effect. In diagram 2 every square is again either occupied or attacked, but in this case every rook is unguarded. Now, in how many different ways can you so place the eight rooks on the board that every square shall be occupied or attacked and no rook ever guarded by another? I do not wish to go into the question of reversals and reflections on this occasion, so that placing the rooks on the other diagonal will count as different, and similarly with other repetitions obtained by turning the board round.

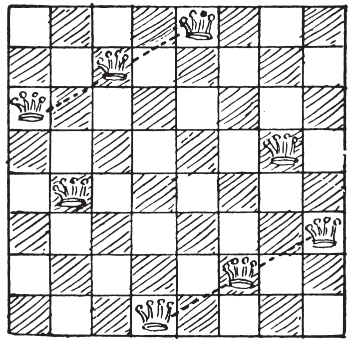
75. THE FOUR LIONS.



The puzzle is to find in how many different ways the four lions may be placed so that there shall never be more than one lion in any row or column. Mere reversals and reflections will not count as different. Thus, regarding the example given, if we place the lions in the other diagonal, it will be considered the same arrangement. For if you hold the second arrangement in front of a mirror or give it a quarter turn, you merely get the first arrangement. It is a simple little puzzle, but requires a certain amount of careful consideration.

76. THE EIGHT QUEENS.

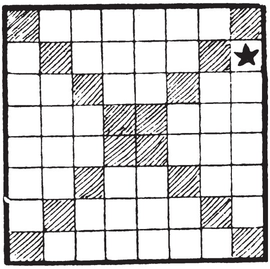
The queen is by far the strongest piece on the chessboard. If you place her on one of the four squares in the centre of the board, she



attacks no fewer than twenty-seven other squares; and if you try to hide her in a corner, she still attacks twenty-one squares. Eight queens may be placed on the board so that no queen attacks another, and it is an old puzzle (first proposed by Nauck in 1850, and it has quite a little literature of its own) to discover in just how many different ways this may be done. I show one way in the diagram, and there are in all twelve of these fundamentally different ways. These twelve produce ninety-two ways if we regard reversals and reflections as different. The diagram is in a way a symmetrical arrangement. If you turn the page upside down, it will reproduce itself exactly; but if you look at it with one of the other sides at the bottom, you get another way that is not identical. Then if you reflect these two ways in a mirror you get two more ways. Now, all the other eleven solutions are non-symmetrical, and therefore each of them may be presented in eight ways by these reversals and reflections. It will thus be seen why the twelve fundamentally different solutions produce only ninety-two arrangements, as I have said, and not ninety-six, as would happen if all twelve were non-symmetrical. It is well to have a clear understanding on the matter of reversals and reflections when dealing with puzzles on the chessboard.

Can the reader place the eight queens on the board so that no queen shall attack another and so that no three queens shall be in a straight line in any oblique direction? Another glance at the diagram will show that this arrangement will not answer the conditions, for in the two directions indicated by the dotted lines there are three queens in a straight line. There is only one of the twelve fundamental ways that will solve the puzzle. Can you find it?

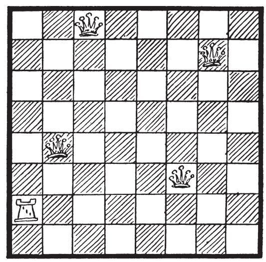
77. THE EIGHT STARS.



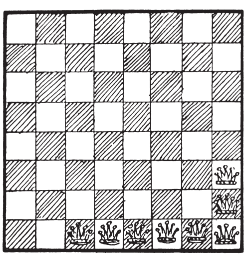
The puzzle in this case is to place eight stars in the diagram so that no star shall be in line with another star horizontally, vertically, or diagonally. One star is already placed, and that must not be moved, so there are only seven for the reader now to place. But you must not place a star on any one of the shaded squares. There is only one way of solving this little puzzle.

78. QUEENS AND BISHOP PUZZLE.

It will be seen that every square of the board is either occupied or attacked. The puzzle is to substitute a bishop for the rook on the same square, and then place the four queens on other squares so that every square shall again be either occupied or attacked.



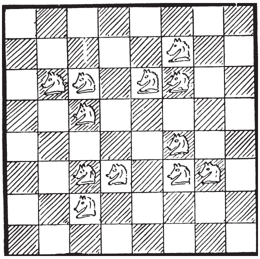
79. THE AMAZONS.



This puzzle is based on one by Captain Turton. Remove three of the queens to other squares so that there shall be eleven squares on the board that are not attacked. The removal of the three queens need not be by 'queen moves.' You may take them up and place them anywhere. There is only one solution.

80. THE KNIGHT-GUARDS.

The knight is the irresponsible low comedian of the chessboard. 'He is a very uncertain, sneaking, and demoralizing rascal,'



says an American writer. 'He can only move two squares, but makes up in the quality of his locomotion for its quantity, for he can spring one square sideways and one forward simultaneously, like a cat; can stand on one leg in the middle of the board and jump to any one of eight squares he chooses; can get on one side of a fence and blackguard three or four men on the other; has an objectionable way of inserting himself in safe places where he can scare the king and compel him to move, and then gobble a queen. For pure cussedness the knight has no equal, and when you chase him out of one hole he skips into another.' Attempts have been made over and over again to obtain a short, simple, and exact definition of the move of the knight—without success. It really consists in moving one square like a rook, and then another square like a bishop—the two operations being done in one leap, so that it does not matter whether the first square passed over is occupied by another piece or not. It is, in fact, the only leaping move in chess. But difficult as it is to define, a child can learn it by inspection in a few minutes.

I have shown in the diagram how twelve knights (the fewest possible that will perform the feat) may be placed on the chessboard so that every square is either occupied or attacked by a knight. Examine every square in turn, and you will find that this is so. Now, the puzzle in this case is to discover what is the smallest possible number of knights that is required in order that every square shall be either occupied or attacked, and every knight protected by another knight. And how would you arrange them? It will be found that of the twelve shown in the diagram only four are thus protected by being a knight's move from another knight.

THE GUARDED CHESSBOARD

On an ordinary chessboard, 8 by 8, every square can be guarded—that is, either occupied or attacked—by 5 queens, the fewest possible. There are exactly 91 fundamentally different arrangements in which no queen attacks another queen. If every queen must attack (or be protected by) another queen, there are at fewest 41 arrangements, and I have recorded some 150 ways in which some of the queens are attacked and some not, but this last case is very difficult to enumerate exactly.

On an ordinary chessboard every square can be guarded by 8 rooks (the fewest possible) in 40,320 ways, if no rook may attack another rook, but it is not known how many of these are fundamentally different. (See solution to No. 73, 'The Eight Rooks.') I have not enumerated the ways in which every rook shall be protected by another rook.

On an ordinary chessboard every square can be guarded by 8 bishops (the fewest possible), if no bishop may attack another bishop. Ten bishops are necessary if every bishop is to be protected.

On an ordinary chessboard every square can be guarded by 12 knights if all but 4 are unprotected. But if every knight must be protected, 14 are necessary. (See No. 79, 'The Knight-Guards.')

Dealing with the queen on n^2 boards generally, where n is less than 8, the following results will be of interest:—

1 queen guards 2^2 board in 1 fundamental way.
1 queen guards 3^2 board in 1 fundamental way.
2 queens guard 4^2 board in 3 fundamental ways (protected).
3 queens guard 4^2 board in 2 fundamental ways (not protected).
3 queens guard 5^2 board in 37 fundamental ways (protected).
3 queens guard 5^2 board in 2 fundamental ways (not protected).
3 queens guard 6^2 board in 1 fundamental way (protected).

4 queens guard 6^2 board in 17 fundamental ways (not protected).
4 queens guard 7^2 board in 5 fundamental ways (protected).
4 queens guard 7^2 board in 1 fundamental way (not protected).

NON-ATTACKING CHESSBOARD ARRANGEMENTS.

We know that n queens may always be placed on a square board of n^2 squares (if n be greater than 3) without any queen attacking another queen. But no general formula for enumerating the number of different ways in which it may be done has yet been discovered; probably it is undiscoverable. The known results are as follows:—

Where $n = 4$ there is 1 fundamental solution and 2 in all.
Where $n = 5$ there are 2 fundamental solutions and 10 in all.
Where $n = 6$ there is 1 fundamental solution and 4 in all.
Where $n = 7$ there are 6 fundamental solutions and 40 in all.
Where $n = 8$ there are 12 fundamental solutions and 92 in all.
Where $n = 9$ there are 46 fundamental solutions.
Where $n = 10$ there are 92 fundamental solutions.
Where $n = 11$ there are 341 fundamental solutions.

Obviously n rooks may be placed without attack on an n^2 board in $|\underline{n}$ ways, but how many of these are fundamentally different I have only worked out in the four cases where n equals 2, 3, 4, and 5. The answers here are respectively 1, 2, 7, and 23.

We can place $2n$-2 bishops on an n^2 board in $2n$ ways. For boards containing 2, 3, 4, 5, 6, 7, 8 squares, on a side there are respectively 1, 2, 3, 6, 10, 20, 36 fundamentally different arrangements. Where n is odd there are $2^{\frac{1}{2}(n-1)}$ such arrangements, each giving 4 by reversals and reflections, and $2n-3-2^{\frac{1}{2}(n-3)}$ giving 8. Where n is even there are $2^{\frac{1}{2}(n-2)}$, each giving 4 by reversals and reflections, and $2n-3-2^{\frac{1}{2}(n-4)}$, each giving 8.

We can place $\frac{1}{2}(n^2+1)$ knights on an n^2 board without attack, when n is odd, in 1 fundamental way; and $\frac{1}{2}n^2$ knights on an n^2 board, when n is even, in 1 fundamental way. In the first case we place all the knights on the same colour as the central square; in the second case we place them all on black, or all on white, squares.

THE TWO PIECES PROBLEM.

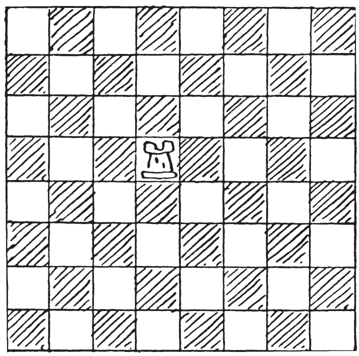
On a board of n^2 squares, two queens, two rooks, two bishops, or two knights can always be placed, irrespective of attack or not, in $\frac{n^4 - n^2}{2}$ ways. The following formulæ will show in how many of these ways the two pieces may be placed with attack and without:—

	With Attack.	Without Attack.
2 Queens	$\frac{5n^3 - 6n^2 + n}{3}$	$\frac{3n^4 - 10n^3 + 9n^2 - 2n}{6}$
2 Rooks	$n^3 - n^2$	$\frac{n^4 - 2n^3 + n^2}{2}$
2 Bishops	$\frac{4n^3 - 6n^2 + 2n}{6}$	$\frac{3n^4 - 4n^3 + 3n^2 - 2n}{6}$
2 Knights	$4n^2 - 12n + 8$	$\frac{n^4 - 9n^2 + 24n}{2}$

DYNAMICAL CHESS PUZZLES

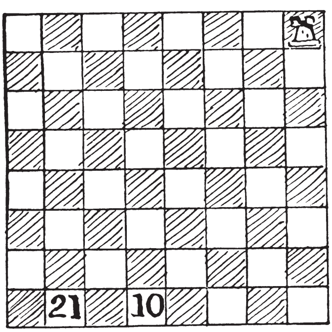
'Push on—keep moving.'
THOS. MORTON: *Cure for the Heartache*.

81. THE ROOK'S TOUR.



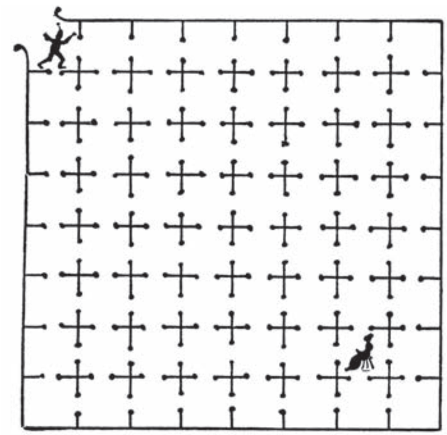
The puzzle is to move the single rook over the whole board, so that it shall visit every square of the board once, and only once, and end its tour on the square from which it starts. You have to do this in as few moves as possible, and unless you are very careful you will take just one move too many. Of course, a square is regarded equally as 'visited' whether you merely pass over it or make it a stopping-place, and we will not quibble over the point whether the original square is actually visited twice. We will assume that it is not.

82. THE ROOK'S JOURNEY.



This puzzle I call 'The Rook's Journey,' because the word 'tour' (derived from a turner's wheel) implies that we return to the point from which we set out, and we do not do this in the present case. We should not be satisfied with a personally conducted holiday tour that ended by leaving us, say, in the middle of the Sahara. The rook here makes twenty-one moves, in the course of which journey it visits every square of the board once and only once, stopping at the square marked 10 at the end of its tenth move, and ending at the square marked 21. Two consecutive moves cannot be made in the same direction—that is to say, you must make a turn after every move.

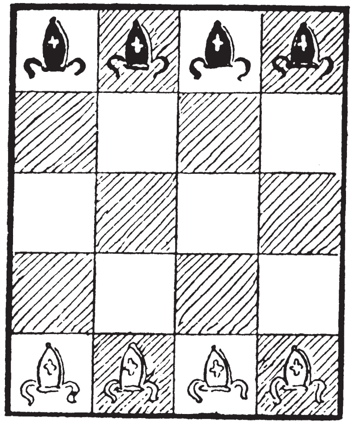
83. THE LANGUISHING MAIDEN.



A wicked baron in the good old days imprisoned an innocent maiden in one of the deepest dungeons beneath the castle moat. It will be seen from our illustration that there were sixty-three cells in the dungeon, all connected by open doors, and the maiden was chained in the cell in which she is shown. Now, a valiant knight, who loved the damsel, succeeded in rescuing her from the enemy. Having gained an entrance to the dungeon at the point where he is seen, he succeeded in reaching the maiden after entering every cell once and only once. Take your pencil and try to trace out such a route. When you have succeeded, then try to discover a route in twenty-two straight paths through the cells. It can be done in this number without entering any cell a second time.

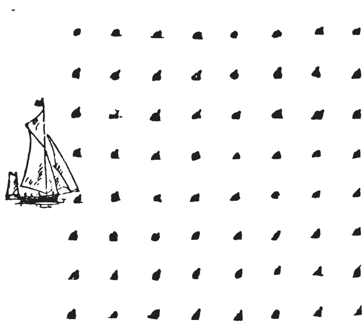
84. A NEW BISHOP'S PUZZLE.

This is quite a fascinating little puzzle. Place eight bishops (four black and four white) on the reduced chessboard, as shown in the illustration. The problem is to make the black bishops change places with the white ones, no bishop ever attacking another of the opposite colour. They must move alternately—first a white, then a black, then a white, and so on. When you have succeeded in doing it at all, try to find the fewest possible moves.



85. THE YACHT RACE.

Now then, ye land-lubbers, hoist your baby-jib-topsails, break out your spinnakers, ease off your balloon sheets, and get your head-sails set!

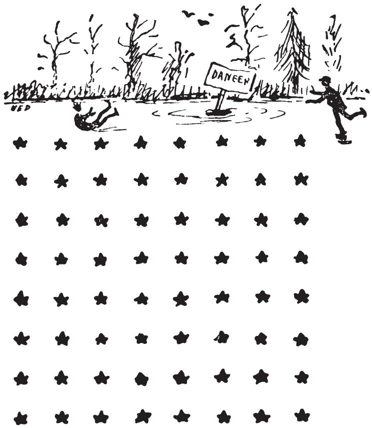


Our race consists in starting from the point at which the yacht is lying in the illustration and touching every one of the sixty-four buoys in fourteen straight courses, returning in the final tack to the buoy from which we start. The seventh course must finish at the buoy from which a flag is flying.

This puzzle will call for a lot of skilful seamanship on account of the sharp angles at which it will occasionally be necessary to tack. The point of a lead pencil and a good nautical eye are all the outfit that we require.

This is difficult, because of the condition as to the flag-buoy, and because it is a re-entrant tour. But again we are allowed those oblique lines.

86. THE SCIENTIFIC SKATER.

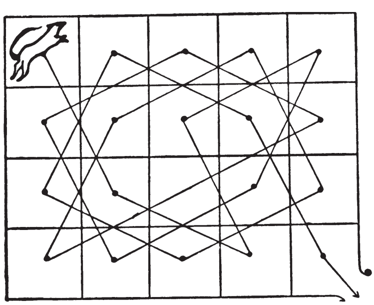


It will be seen that this skater has marked on the ice sixty-four points or stars, and he proposes to start from his present position near the corner and enter every one of the points in fourteen straight lines. How will he do it? Of course there is no objection to his passing over any point more than once, but his last straight stroke must bring him back to the position from which he started.

It is merely a matter of taking your pencil and starting from the spot on which the skater's foot is at present resting, and striking out all the stars in fourteen continuous straight lines, returning to the point from which you set out.

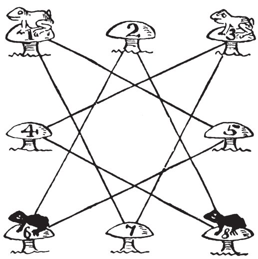
87. THE GREYHOUND PUZZLE.

In this puzzle the twenty kennels do not communicate with one another by doors, but are divided off by a low wall. The solitary occupant is the greyhound which lives in the kennel in the top left-hand corner. When he is allowed his liberty he



has to obtain it by visiting every kennel once and only once in a series of knight's moves, ending at the bottom right-hand corner, which is open to the world. The lines in the above diagram show one solution. The puzzle is to discover in how many different ways the greyhound may thus make his exit from his corner kennel.

88. THE FOUR FROGS.



In the illustration we have eight toadstools, with white frogs on 1 and 3 and black frogs on 6 and 8. The puzzle is to move one frog at a time, in any order, along one of the straight lines from toadstool to toadstool, until they have exchanged places, the white frogs being left on 6 and 8 and the black ones on 1 and 3. If you use four counters on a simple diagram, you will find this quite easy, but it is a little more puzzling to do it in only seven plays, any number of successive moves by one frog counting as one play. Of course, more than one frog cannot be on a toadstool at the same time.

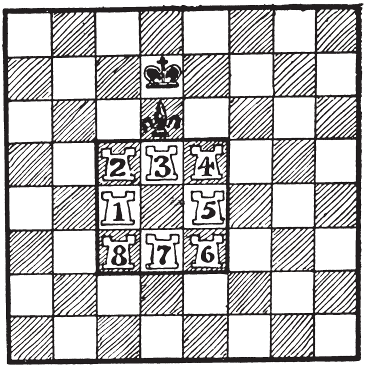
VARIOUS CHESS PUZZLES

'Chesse-play is a good and wittie exercise of the minde for some kinde of men.'
BURTON'S *Anatomy of Melancholy*.

89. COUNTING THE RECTANGLES.

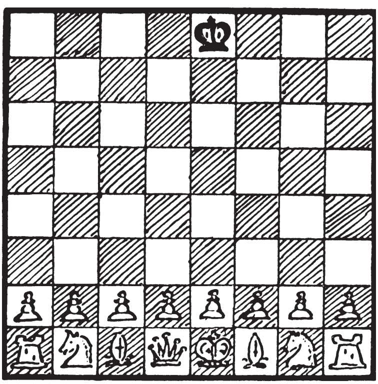
Can you say correctly just how many squares and other rectangles the chessboard contains? In other words, in how great a number of different ways is it possible to indicate a square or other rectangle enclosed by lines that separate the squares of the board?

90. THE ROOKERY.



The White rooks cannot move outside the little square in which they are enclosed except on the final move, in giving checkmate. The puzzle is how to checkmate Black in the fewest possible moves with No. 8 rook, the other rooks being left in numerical order round the sides of their square with the break between 1 and 7.

91. THE FORSAKEN KING.

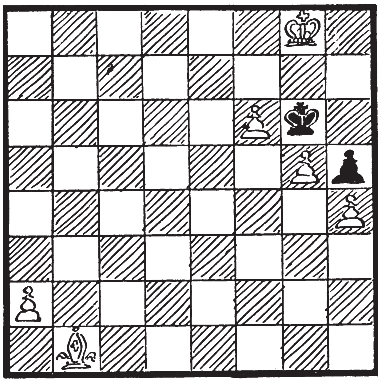


Set up the position shown in the diagram. Then the condition of the puzzle is—White to play and checkmate insixmoves.Notwithstanding the complexities, I will show how the manner of play may be condensed into quite a few lines, merely stating here that the first two moves of White cannot be varied.

92. CHECKMATE!

Strolling into one of the rooms of a London club, I noticed a position left by two players who had gone. This position is shown in the diagram. It is evident that White has checkmated Black. But how did he do it? That is the puzzle.

Black



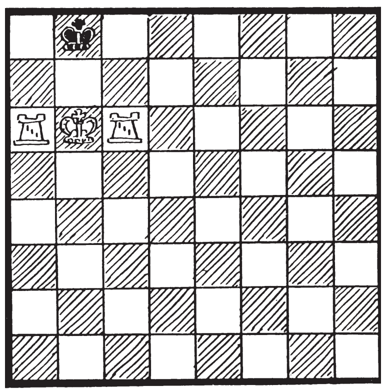
White

(See No. 92. CHECKMATE!)

93. ANCIENT CHINESE PUZZLE.

My next puzzle is supposed to be Chinese, many hundreds of years old, and never fails to interest. White to play and mate, moving each of the three pieces once, and once only.

Black



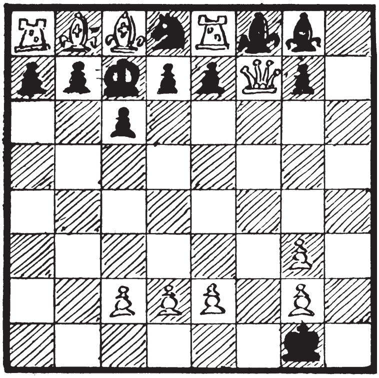
White

94. THE MONSTROSITY.

One Christmas Eve I was travelling by rail to a little place in one of the southern counties. The compartment was very full, and the passengers were wedged in very tightly. My neighbour in one of the corner seats was closely studying a position set up on one of those little folding chessboards that can be carried conveniently in the pocket, and I could scarcely avoid looking at it myself. Here is the position:—

My fellow-passenger suddenly turned his head and caught the look of bewilderment on my face.

Black



White

'Do you play chess?' he asked.

'Yes, a little. What is that? A problem?'

'Problem? No; a game.'

'Impossible!' I exclaimed rather rudely. 'The position is a perfect monstrosity!'

He took from his pocket a postcard and handed it to me. It bore an address at one side and on the other the words '43. K to Kt 8.'

'It is a correspondence game.' he exclaimed. 'That is my friend's last move, and I am considering my reply.'

'But you really must excuse me; the position seems utterly impossible. How on earth, for example—'

'Ah!' he broke in smilingly. 'I see; you are a beginner; you play to win.'

'Of course you wouldn't play to lose or draw!'

He laughed aloud.

'You have much to learn. My friend and myself do not play for results of that antiquated kind. We seek in chess the wonderful, the whimsical, the weird. Did you ever see a position like that?'

I inwardly congratulated myself that I never had.

'That position, sir, materializes the sinuous evolvements and syncretic, synthetic, and synchronous concatenations of two cerebral individualities. It is the product of an amphoteric and intercalatory interchange of—'

'Have you seen the evening paper, sir?' interrupted the man opposite, holding out a newspaper. I noticed on the margin beside his thumb some pencilled writing. Thanking him, I took the paper and read—'Insane, but quite harmless. He is in my charge.'

After that I let the poor fellow run on in his wild way until both got out at the next station.

But that queer position became fixed indelibly in my mind, with Black's last move 43. K to Kt 8; and a short time afterwards I found it actually possible to arrive at such a position in forty-three moves. Can the reader construct such a sequence? How did White get his rooks and king's bishop into their present positions, considering Black can never have moved his king's bishop? No odds were given, and every move was perfectly legitimate.

CROSSING RIVER PROBLEMS

'My boat is on the shore.'

BYRON.

This is a mediæval class of puzzles. Probably the earliest example was by Abbot Alcuin, who was born in Yorkshire in 735 and died at Tours in 804. And everybody knows the story of the man with the wolf, goat, and basket of cabbages whose boat would only take one of the three at a time with the man himself. His difficulties arose from his being unable to leave the wolf alone with the goat, or the goat alone with the cabbages. These puzzles were considered by Tartaglia and Bachet, and have been later investigated by Lucas, De Fonteney, Delannoy, Tarry, and others. In the puzzles I give there will be found one or two new conditions which add to the complexity somewhat. I also include a pulley problem that practically involves the same principles.

95. CROSSING THE STREAM.

During a country ramble Mr. and Mrs. Softleigh found themselves in a pretty little dilemma. They had to cross a stream in a small boat which was capable of carrying only 150 lbs. weight. But Mr. Softleigh and his wife each weighed exactly 150 lbs., and each of their sons weighed 75 lbs. And then there was the dog, who could not be induced on any terms to swim. On the principle of 'ladies first,' they at once sent Mrs. Softleigh over; but this was a stupid oversight, because she had to come back again with the boat, so nothing was gained by that operation. How did they all succeed in getting across?

96. CROSSING THE RIVER AXE.

Many years ago, in the days of the smuggler known as 'Rob Roy of the West,' a piratical band buried on the coast of South Devon a quantity of treasure which was, of course, abandoned by them in the usual inexplicable way. Some time afterwards its whereabouts was discovered by three countrymen, who visited the spot one night and divided the spoil between them, Giles taking treasure to the value of £800, Jasper £500 worth, and Timothy £300 worth. In returning they had to cross the river Axe at a point where they had left a small boat in readiness. Here, however, was a difficulty they had not anticipated. The boat would only carry two men, or one man and a sack, and they had so little confidence in one another that no person could be left alone on the land or in the boat with more than his share of the spoil, though two persons (being a check on each other) might be left with more than their shares. The puzzle is to show how they got over the river in the fewest possible crossings, taking their treasure with them. No tricks, such as ropes, 'flying bridges,' currents, swimming, or similar dodges, may be employed.

97. FIVE JEALOUS HUSBANDS.

During certain local floods five married couples found themselves surrounded by water, and had to escape from their unpleasant position in a boat that would only hold three persons at a time. Every husband was so jealous that he would not allow his wife to be in the boat or on either bank with another man (or with other men) unless he was himself present. Show the quickest way of getting these five men and their wives across into safety.

Call the men A, B, C, D, E, and their respective wives a, b, c, d, e. To go over and return counts as two crossings. No tricks such as ropes, swimming, currents, etc., are permitted.

98. THE FOUR ELOPEMENTS.

Colonel B – was a widower of a very taciturn disposition. His treatment of his four daughters was unusually severe, almost cruel, and they not unnaturally felt disposed to resent it. Being charming girls with every virtue and many accomplishments, it is not surprising that each had a fond admirer. But the father forbade the young men to call at his house, intercepted all letters, and placed his daughters under stricter supervision than ever. But love, which scorns locks and keys and garden walls, was equal to the occasion, and the four youths conspired together and planned a general elopement.

At the foot of the tennis lawn at the bottom of the garden ran the silver Thames, and one night, after the four girls had been safely conducted from a dormitory window to terra firma, they all crept softly down to the bank of the river, where a small boat belonging to the Colonel was moored. With this they proposed to cross to the opposite side and make their way to a lane where conveyances were waiting to carry them in their flight. Alas! here at the water's brink their difficulties already began.

The young men were so extremely jealous that not one of them would allow his prospective bride to remain at any time in the company of another man, or men, unless he himself were present also. Now, the boat would only hold two persons, though it could, of course, be rowed by one, and it seemed impossible that the four couples would ever get across. But midway in the stream was a small island, and this seemed to present a way out of the difficulty, because a person or persons could be left there while the boat was rowed back or to the opposite shore. If they had been prepared for their difficulty they could have easily worked out a solution to the little poser at any other time. But they were now so hurried and excited in their flight that the confusion they soon got into was exceedingly amusing—or would have been to any one except themselves.

As a consequence they took twice as long and crossed the river twice as often as was really necessary. Meanwhile, the Colonel, who was a very light sleeper, thought he heard a splash of oars. He quickly raised the alarm among his household, and the young ladies were found to be missing. Somebody was sent to the police-station, and a number of officers soon aided in the pursuit of the fugitives, who, in consequence of that delay in crossing the river, were quickly overtaken. The four girls returned sadly to their homes, and afterwards broke off their engagements in disgust.

For a considerable time it was a mystery how the party of eight managed to cross the river in that little boat without any girl being ever left with a man, unless her betrothed was also present. The favourite method is to take eight counters or pieces of cardboard and mark them A, B, C, D, a, b, c, d, to represent the four men and their prospective brides, and carry them from one side of a table to the other in a matchbox (to represent the boat), a penny being placed in the middle of the table as the island.

Readers are now asked to find the quickest method of getting the party across the river. How many passages are necessary from land to land? By ‘land’ is understood either shore or island. Though the boat would not necessarily call at the island every time of crossing, the possibility of its doing so must be provided for. For example, it would not do for a man to be alone in the boat (though it were understood that he intended merely to cross from one bank to the opposite one) if there happened to be a girl alone on the island other than the one to whom he was engaged.

99. STEALING THE CASTLE TREASURE.

The ingenious manner in which a box of treasure, consisting principally of jewels and precious stones, was stolen from Gloomhurst Castle has been handed down as a tradition in the De Gourney family. The thieves consisted of a man, a youth, and a small boy, whose only mode of escape with the box of treasure was by means of a high window. Outside the window was fixed a pulley, over which ran a rope with a basket at each end. When one basket was on the ground the other was at the window. The rope was so disposed that the persons in the basket could neither help themselves by means of it nor receive help from others. In short, the only way the baskets could be used was by placing a heavier weight in one than in the other.

Now, the man weighed 195 lbs., the youth 105 lbs., the boy 90 lbs., and the box of treasure 75 lbs. The weight in the descending basket could not exceed that in the other by more than 15 lbs. without causing a descent so rapid as to be most dangerous to a human being, though it would not injure the stolen property. Only two persons, or one person and the treasure, could be placed in the same basket at one time. How did they all manage to escape and take the box of treasure with them?

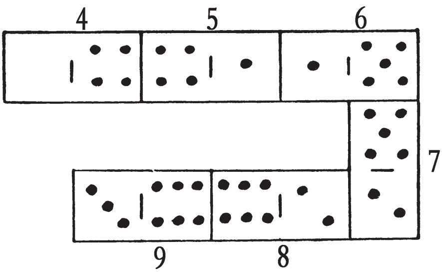
The puzzle is to find the shortest way of performing the feat, which in itself is not difficult. Remember, a person cannot help himself by hanging on to the rope, the only way being to go down ‘with a bump,’ with the weight in the other basket as a counterpoise.

PROBLEMS CONCERNING GAMES

'The little pleasure of the game.'
MATTHEW PRIOR.

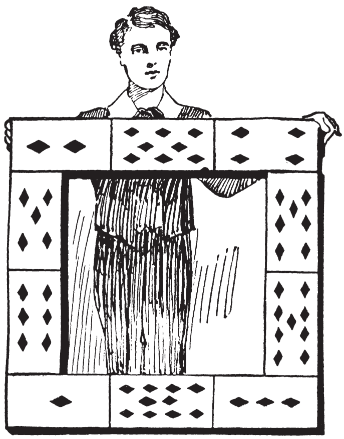
Every game lends itself to the propounding of a variety of puzzles. They can be made, as we have seen, out of the chessboard and the peculiar moves of the chess pieces. I will now give just a few examples of puzzles with playing cards and dominoes, and also go out of doors and consider one or two little posers in the cricket field, at the football match, and the horse race and motor-car race.

100. DOMINOES IN PROGRESSION.



It will be seen that I have played six dominoes, in the illustration, in accordance with the ordinary rules of the game, 4 against 4, 1 against 1, and so on, and yet the sum of the spots on the successive dominoes, 4, 5, 6, 7, 8, 9, are in arithmetical progression; that is, the numbers taken in order have a common difference of 1. In how many different ways may we play six dominoes, from an ordinary box of twenty-eight, so that the numbers on them may lie in arithmetical progression? We must always play from left to right, and numbers in decreasing arithmetical progression (such as 9, 8, 7, 6, 5, 4) are not admissible.

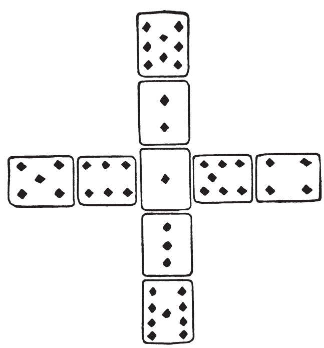
101. THE CARD FRAME PUZZLE.



In the illustration we have a frame constructed from the ten playing cards, ace to ten of diamonds. The children who made it wanted the pips on all four sides to add up alike, but they failed in their attempt and gave it up as impossible. It will be seen that the pips in the top row, the bottom row, and the left-hand side all add up 14, but the right-hand side sums to 23. Now, what they were trying to do is quite possible. Can you rearrange the ten cards in the same formation so that all four sides shall add up alike? Of course they need not add up 14, but any number you choose to select.

102. THE CROSS OF CARDS.

In this case we use only nine cards—the ace to nine of diamonds. The puzzle is to arrange them in the form of a cross, exactly in the way shown in the illustration, so that the pips in the vertical bar and in the horizontal bar add up alike. In the example given it will be found that both directions add up 23. What I want to know is, how many different ways are there of



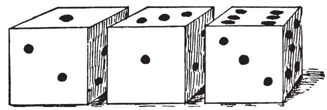
rearranging the cards in order to bring about this result? It will be seen that, without affecting the solution, we may exchange the 5 with the 6, the 5 with the 7, the 8 with the 3, and so on. Also we may make the horizontal and the vertical bars change places. But such obvious manipulations as these are not to be regarded as different solutions. They are all mere variations of one fundamental solution. Now, how many of these fundamentally different solutions are there? The pips need not, of course, always add up 23.

103. 'STRAND' PATIENCE.

Make two piles of cards as follows: 9 D, 8 S, 7 D, 6 S, 5 D, 4 S, 3 D, 2 S, 1 D, and 9 H, 8 C, 7 H, 6 C, 5 H, 4 C, 3 H, 2 C, 1 H, with the 9 of diamonds at the bottom of one pile and the 9 of hearts at the bottom of the other. The point is to exchange the spades with the clubs, so that the diamonds and clubs are still in numerical order in one pile and the hearts and spades in the other. There are four vacant spaces in addition to the two spaces occupied by the piles, and any card may be laid on a space, but a card can only be laid on another of the next higher value—an ace on a two, a two on a three, and so on. Patience is required to discover the shortest way of doing this. When there are four vacant spaces you can pile four cards in seven moves, with only three spaces you can pile them in nine moves, and with two spaces you cannot pile more than two cards. When you have a grasp of these and similar facts you will be able to remove a number of cards bodily and write down 7, 9, or whatever the number of moves may be. The gradual shortening of play is fascinating, and first attempts are surprisingly lengthy.

104. A TRICK WITH DICE.

Here is a neat little trick with three dice. I ask you to throw the dice without my seeing them. Then I tell you to multiply the



points of the first die by 2 and add 5; then multiply the result by 5 and add the points of the second die; then multiply the result by 10 and add the points of the third die. You then give me the total, and I can at once tell you the points thrown with the three dice. How do I do it? As an example, if you threw 1, 3, and 6, as in the illustration, the result you would give me would be 386, from which I could at once say what you had thrown.

105. THE VILLAGE CRICKET MATCH.

In a cricket match, Dingley Dell v. All Muggleton, the latter had the first innings. Mr. Dumkins and Mr. Podder were at the wickets, when the wary Dumkins made a splendid late cut, and Mr. Podder called on him to run. Four runs were apparently completed, but the vigilant umpires at each end called, 'three short,' making six short runs in all. What number did Mr. Dumkins score? When Dingley Dell took their turn at the wickets their champions were Mr. Luffey and Mr. Struggles. The latter made a magnificent off-drive, and invited his colleague to 'come along,' with the result that the observant spectators applauded them for what was supposed to have been three sharp runs. But the umpires declared that there had been two short runs at each end—four in all. To what extent, if any, did this manoeuvre increase Mr. Struggles's total?

PUZZLE GAMES

'He that is beaten may be said
To lie in honour's truckle bed.'

HUDIBRAS.

It may be said generally that a game is a contest of skill for two or more persons, into which we enter either for amusement or to win a prize. A puzzle is something to be done or solved by the individual. For example, if it were possible for us so to master the complexities of the game of chess that we could be assured of always winning with the first or second move, as the case might be, or of always drawing, then it would cease to be a game and would become a puzzle. Of course among the young and uninformed, when the correct winning play is not understood, a puzzle may well make a very good game. Thus there is no doubt children will continue to play 'Noughts and Crosses,' though between two players who both thoroughly understand the play, every game should be drawn. Neither player could ever win except through the blundering of his opponent.

The examples that I give in this class are apparently games, but, since I show in every case how one player may win if he only play correctly, they are in reality puzzles. Their interest, therefore, lies in attempting to discover the leading method of play.

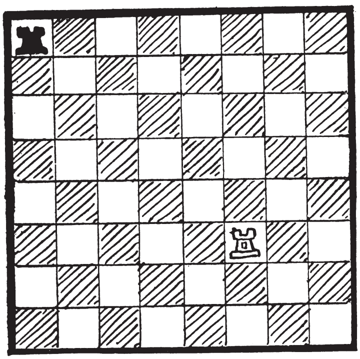
106. THE PEBBLE GAME.

Here is an interesting little puzzle game that I used to play with an acquaintance on the beach at Slocomb-on-Sea. Two players place an odd number of pebbles, we will say fifteen, between them. Then each takes in turn one, two, or three pebbles (as he chooses), and the winner is the one who gets the odd number.

Thus, if you get seven and your opponent eight, you win. If you get six and he gets nine, he wins. Ought the first or second player to win, and how? When you have settled the question with fifteen pebbles try again with, say, thirteen.

107. THE TWO ROOKS.

Black

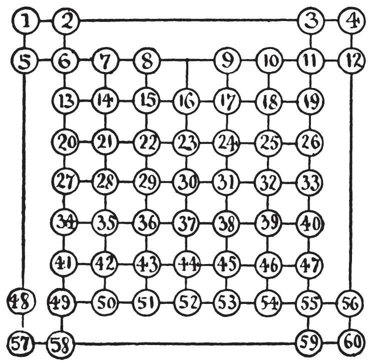


White

This is a puzzle game for two players. Each player has a single rook. The first player places his rook on any square of the board that he may choose to select, and then the second player does the same. They now play in turn, the point of each play being to capture the opponent's rook. But in this game you cannot play through a line of attack without being captured. That is to say, if in the diagram it is Black's turn to play, he cannot move his rook to his king's knight's square, or to his king's rook's square, because he would enter the 'line of fire' when passing his king's bishop's square. For the same reason he cannot move to his queen's rook's seventh or eighth squares. Now, the game can never end in a draw. Sooner or later one of the rooks must fall, unless, of course, both players commit the absurdity of not trying to win. The trick of winning is ridiculously simple when you know it. Can you solve the puzzle?

108. PUSS IN THE CORNER.

This variation of the last puzzle is also played by two persons. One puts a counter on No. 6, and the other puts one on No. 55, and they play alternately by removing the counter to any other number in a line. If your opponent moves at any time



on to one of the lines you occupy, or even crosses one of your lines, you immediately capture him and win. We will take an illustrative game.

A moves from 55 to 52; B moves from 6 to 13; A advances to 23; B goes to 15; A retreats to 26; B retreats to 13; A advances to 21; B retreats to 2; A advances to 7; B goes to 3; A moves to 6; B must now go to 4; A establishes himself at 11, and B must be captured next move because he is compelled to cross a line on which A stands. Play this over and you will understand the game directly. Now, the puzzle part of the game is this: Which player should win, and how many moves are necessary?

109. A MATCH MYSTERY.

Here is a little game that is childishly simple in its conditions. But it is well worth investigation.

Mr. Stubbs pulled a small table between himself and his friend, Mr. Wilson, and took a box of matches, from which he counted out thirty.

'Here are thirty matches,' he said. 'I divide them into three unequal heaps. Let me see. We have 14, 11, and 5, as it happens. Now, the two players draw alternately any number from any one heap, and he who draws the last match loses the game. That's all! I will play with you, Wilson. I have formed the heaps, so you have the first draw.'

'As I can draw any number,' Mr. Wilson said, 'suppose I exhibit my usual moderation and take all the 14 heap.'

'That is the worst you could do, for it loses right away. I take 6 from the 11, leaving two equal heaps of 5, and to leave two equal heaps is a certain win (with the single exception of 1, 1), because whatever you do in one heap I can repeat in the other. If you leave 4 in one heap, I leave 4 in the other. If you then leave 2 in one heap, I leave 2 in the other. If you leave only 1 in one heap, then I take all the other heap. If you take all one heap, I take all but one in the other. No, you must never leave two heaps, unless they are equal heaps and more than 1, 1. Let's begin again.'

'Very well, then,' said Mr. Wilson. 'I will take 6 from the 14, and leave you 8, 11, 5.'

Mr. Stubbs then left 8, 11, 3; Mr. Wilson, 8, 5, 3; Mr. Stubbs, 6, 5, 3; Mr. Wilson,4, 5, 3; Mr. Stubbs, 4, 5, 1; Mr. Wilson, 4, 3, 1; Mr. Stubbs, 2, 3, 1; Mr. Wilson, 2, 1, 1; which Mr. Stubbs reduced to 1, 1, 1.

'It is now quite clear that I must win,' said Mr. Stubbs, 'because you must take 1, and then I take 1, leaving you the last match. You never had a chance. There are just thirteen different ways in which the matches may be grouped at the start for a certain win. In fact, the groups selected, 14, 11, 5, are a certain win, because for whatever your opponent may play there is another winning group you can secure, and so on and on down to the last match.'

MAGIC SQUARE PROBLEMS

'By magic numbers.'
CONGREVE, *The Mourning Bride*.

This is a very ancient branch of mathematical puzzledom, and it has an immense, though scattered, literature of its own. In their simple form of consecutive whole numbers arranged in a square so that every column, every row, and each of the two long diagonals shall add up alike, these magic squares offer three main lines of investigation: Construction, Enumeration, and Classification. Of recent years many ingenious methods have been devised for the construction of magics, and the law of their formation is so well understood that all the ancient mystery has evaporated and there is no longer any difficulty in making squares of any dimensions. Almost the last word has been said on this subject.

The question of the enumeration of all the possible squares of a given order stands just where it did over two hundred years ago. Everybody knows that there is only one solution for the third order, three cells by three; and Frénicle published in 1693 diagrams of all the arrangements of the fourth order—880 in number—and his results have been verified over and over again. I may here refer to the general solution for this order, for numbers not necessarily consecutive, by E. Bergholt in *Nature*, May 26, 1910, as it is of the greatest importance to students of this subject. The enumeration of the examples of any higher order is a completely unsolved problem.

As to classification, it is largely a matter of individual taste—perhaps an æsthetic question, for there is beauty in the law and order of numbers. However, lovers of these things seem somewhat agreed that Nasik magic squares (so named by Mr.

Frost, a student of them, after the town in India where he lived, and also called Diabolique and Pandiagonal) and Associated magic squares are of special interest, so I will just explain what these are for the benefit of the novice.

SIMPLE

1	12	14	7
4	15	9	6
13	2	8	11
16	5	3	10

SEMI-NASIK

1	14	12	7
4	15	9	6
13	2	8	11
16	3	5	10

ASSOCIATED

1	14	12	7
8	11	13	2
15	4	6	9
10	5	3	16

NASIK

1	14	7	12
15	4	9	6
10	5	16	3
8	11	2	13

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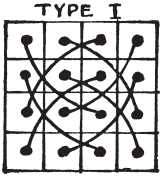

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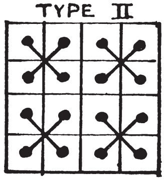

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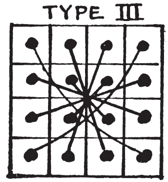

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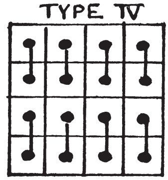

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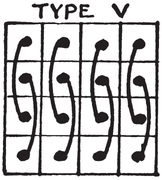

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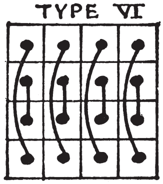
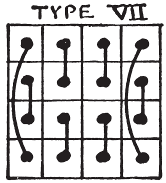


TYPE VII



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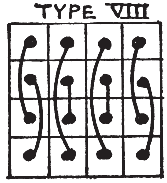

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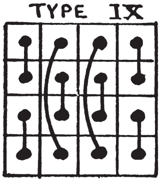

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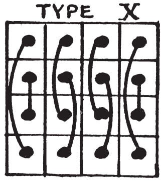

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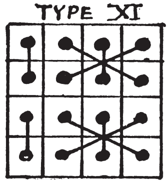

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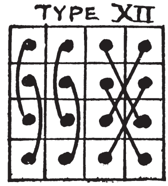


The first example is that of a Simple square that fulfils the simple conditions and no more. The second example is a Semi-Nasik, which has the additional property that the opposite short diagonals of two cells each together sum to 34. Thus, 14 + 4 + 11 + 5 = 34 and 12 + 6 + 13 + 3 = 34. The third example is not only Semi-Nasik but also Associated, because in it every number, if added to the number that is equidistant, in a straight line, from the centre gives 17. Thus, 1 + 16, 2 + 15, 3 + 14, etc. The fourth example, considered the most 'perfect' of all, is a Nasik. Here all the broken diagonals sum to 34. Thus, for example, 15 + 14 + 2 + 3, and 10 + 4 + 7 + 13, and 15 + 5 + 2 + 12. As a consequence, its properties are such that if you repeat the square in all directions you may mark off a square, 4 × 4, wherever you please, and it will be magic.

The following table not only gives a complete enumeration under the four forms described, but also a classification under the twelve graphic types indicated in the diagrams. The dots at the end of each line represent the relative positions of those complementary pairs, 1 + 16, 2 + 15, etc., which sum to 17. For example, it will be seen that the first and second magic squares given are of Type VI., that the third square is of Type III., and that the fourth is of Type I. Edouard Lucas indicated these types, but he dropped exactly half of them and did not attempt the classification.

NASIK	(Type I.)			48
SEMI-NASIK	(Type II., Transpositions of Nasik) .		48	
”	(Type III., Associated)		48	
”	(Type IV.) . . .	96		
”	(Type V.) . . .	96	192	
”	(Type VI.) . . .		96	384
SIMPLE.	(Type VI.) . . .		208	

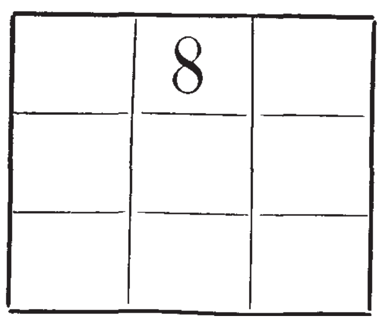
”	(Type VII.) . . .	56		
”	(Type VIII.). . .	56		
”	(Type IX.) . . .	56		
”	(Type X.) . . .	56	224	
”	(Type XI.) . . .	8		
”	(Type XII.) . . .	8	16	448
				880

It is hardly necessary to say that every one of these squares will produce seven others by mere reversals and reflections, which we do not count as different. So that there are 7,040 squares of this order, 880 of which are fundamentally different.

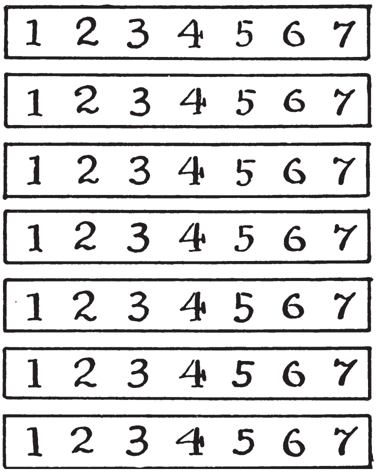
An infinite variety of puzzles may be made introducing new conditions into the magic square.

110. THE TROUBLESOME EIGHT.

Nearly everybody knows that a ‘magic square’ is an arrangement of numbers in the form of a square so that every row, every column, and each of the two long diagonals adds up alike. For example, you would find little difficulty in merely placing a different number in each of the nine cells in the illustration so that the rows, columns, and diagonals shall all add up 15. And at your first attempt you will probably find that you have an 8 in one of the corners. The puzzle is to construct the magic square, under the same conditions, with the 8 in the position shown.



111. THE MAGIC STRIPS.



I happened to have lying on my table a number of strips of cardboard, with numbers printed on them from 1 upwards in numerical order. The idea suddenly came to me, as ideas have a way of unexpectedly coming, to make a little puzzle of this. I wonder whether many readers will arrive at the same solution that I did.

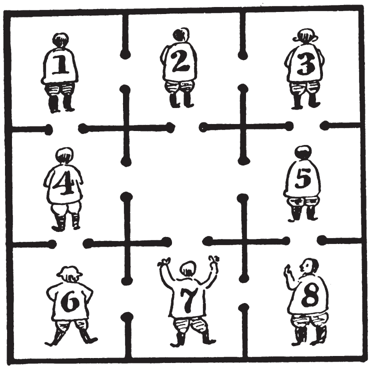
Take seven strips of cardboard and lay them together as above. Then write on each of them the numbers 1, 2, 3, 4, 5, 6, 7, as shown, so that the numbers shall form seven rows and seven columns.

Now, the puzzle is to cut these strips into the fewest possible pieces so that they may be placed together and form a magic square, the seven rows, seven columns, and two diagonals adding up the same number. No figures may be turned upside down or placed on their sides—that is, all the strips must lie in their original direction.

Of course you could cut each strip into seven separate pieces, each piece containing a number, and the puzzle would then be very easy, but I need hardly say that forty-nine pieces is a long way from being the fewest possible.

112. EIGHT JOLLY GAOL BIRDS.

The illustration shows the plan of a prison of nine cells all communicating with one another by doorways. The eight prisoners have their numbers on their backs, and any one of them is allowed to exercise himself in whichever cell may

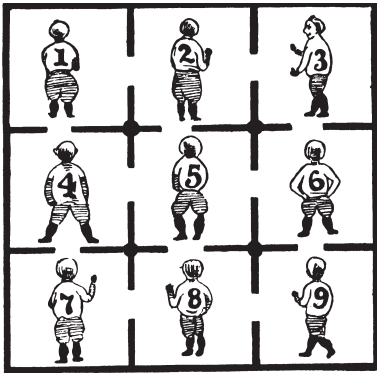


happen to be vacant, subject to the rule that at no time shall two prisoners be in the same cell. The merry monarch in whose dominions the prison was situated offered them special comforts one Christmas Eve if, without breaking that rule, they could so place themselves that their numbers should form a magic square.

Now, prisoner No. 7 happened to know a good deal about magic squares, so he worked out a scheme and naturally selected the method that was most expeditious—that is, one involving the fewest possible moves from cell to cell. But one man was a surly, obstinate fellow (quite unfit for the society of his jovial companions), and he refused to move out of his cell or take any part in the proceedings. But No. 7 was quite equal to the emergency, and found that he could still do what was required in the fewest possible moves without troubling the brute to leave his cell. The puzzle is to show how he did it and, incidentally, to discover which prisoner was so stupidly obstinate. Can you find the fellow?

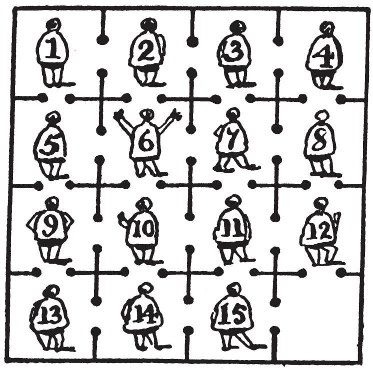
113. NINE JOLLY GAOL BIRDS.

Shortly after the episode recorded in the last puzzle occurred, a ninth prisoner was placed in the vacant cell, and the merry monarch then offered them all complete liberty on the following strange conditions. They were required so to rearrange themselves in the cells that their numbers formed a magic square without their movements causing any two of them ever to be in the same cell together, except that at the start one man was allowed to be placed on the shoulders of another man,



and thus add their numbers together, and move as one man. For example, No. 8 might be placed on the shoulders of No. 2, and then they would move about together as 10. The reader should seek first to solve the puzzle in the fewest possible moves, and then see that the man who is burdened has the least possible amount of work to do.

114. THE SPANISH DUNGEON.

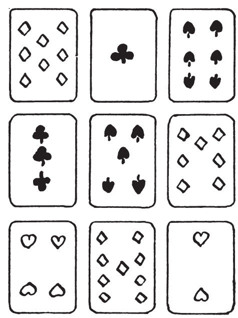


Not fifty miles from Cadiz stood in the middle ages a castle, all traces of which have for centuries disappeared. Among other interesting features, this castle contained a particularly unpleasant dungeon divided into sixteen cells, all communicating with one another, as shown in the illustration. Now, the governor was a merry wight, and very fond of puzzles withal. One day he went to the dungeon and said to the prisoners, 'By my halidame!' (or its equivalent in Spanish) 'you shall all be set free if you can solve this puzzle. You must so arrange yourselves in the sixteen cells that the numbers on your backs shall form a magic square in which every column, every row, and each of the two diagonals shall add up the same. Only remember this: that in no case may two of you ever be together in the same cell.'

One of the prisoners, after working at the problem for two or three days, with a piece of chalk, undertook to obtain the liberty of himself and his fellow-prisoners if they would follow his directions and move through the doorway from cell to cell in the order in which he should call out their numbers.

He succeeded in his attempt, and, what is more remarkable, it would seem from the account of his method recorded in the ancient manuscript lying before me, that he did so in the fewest possible moves. The reader is asked to show what these moves were.

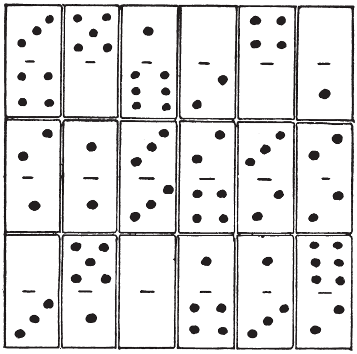
115. CARD MAGIC SQUARES.



Take an ordinary pack of cards and throw out the twelve court cards. Now, with nine of the remainder (different suits are of no consequence) form the above magic square. It will be seen that the pips add up fifteen in every row in every column, and in each of the two long diagonals. The puzzle is with the remaining cards (without disturbing this arrangement) to form three more such magic squares, so that each of the four shall add up to a different sum. There will, of course, be four cards in the reduced pack that will not be used. These four may be any that you choose. It is not a difficult puzzle, but requires just a little thought.

116. THE EIGHTEEN DOMINOES.

The illustration shows eighteen dominoes arranged in the form of a square so that the pips in every one of the six columns, six rows, and two long diagonals add up 13. This is the smallest summation possible with any selection of dominoes from an ordinary box of twenty-eight. The greatest possible summation is 23, and a solution for this number



may be easily obtained by substituting for every number its complement to 6. Thus for every blank substitute a 6, for every 1 a 5, for every 2 a 4, for 3 a 3, for 4 a 2, for 5 a 1, and for 6 a blank. But the puzzle is to make a selection of eighteen dominoes and arrange them (in exactly the form shown) so that the summations shall be 18 in all the fourteen directions mentioned.

SUBTRACTING, MULTIPLYING, AND DIVIDING MAGICS

Although the adding magic square is of such great antiquity, curiously enough the multiplying magic does not appear to have been mentioned until the end of the eighteenth century, when it was referred to slightly by one writer and then forgotten until I revived it in *Tit-Bits* in 1897. The dividing magic was apparently first discussed by me in *The Weekly Dispatch* in June 1898. The subtracting magic is here introduced for the first time. It will now be convenient to deal with all four kinds of magic squares together.

ADDING

8	1	6
3	5	7
4	9	2

SUBTRACTING

2	1	4
3	5	7
6	9	8

MULTIPLYING

12	1	18
9	6	4
2	36	3

DIVIDING

3	1	2
9	6	4
18	36	12

In these four diagrams we have examples in the third order of adding, subtracting, multiplying, and dividing squares. In the first the constant, 15, is obtained by the addition of the rows, columns, and two diagonals. In the second case you get the constant, 5, by subtracting the first number in a line from the second, and the result from the third. You can, of course, perform the operation in either direction; but, in order to avoid negative numbers, it is more convenient simply to deduct the middle number from the sum of the two extreme numbers. This is, in effect, the same thing. It will be seen that the constant of the adding square is *n* times that of the subtracting square derived from it, where *n* is the number of cells in the side of square. And the manner of derivation here is simply to reverse the two diagonals. Both squares are 'associated'—a term I have explained in the introductory article to this department.

The third square is a multiplying magic. The constant, 216, is obtained by multiplying together the three numbers in any line. It is 'associated' by multiplication, instead of by addition. It is here necessary to remark that in an adding square it is not essential that the nine numbers should be consecutive. Write down any nine numbers in this way—

1	3	5
4	6	8
7	9	11

so that the horizontal differences are all alike and the vertical differences also alike (here 2 and 3), and these numbers will form an adding magic square. By making the differences 1 and 3 we, of course, get consecutive numbers—a particular case, and nothing more. Now, in the case of the multiplying square we must take these numbers in geometrical instead of arithmetical progression, thus—

1	3	9
2	6	18
4	12	36

Here each successive number in the rows is multiplied by 3, and in the columns by 2. Had we multiplied by 2 and 8 we should get the regular geometrical progression, 1, 2, 4, 8, 16, 32, 64, 128, and 256, but I wish to avoid high numbers. The numbers are arranged in the square in the same order as in the adding square. The fourth diagram is a dividing magic square. The constant 6 is here obtained by dividing the second number in a line by the first (in either direction) and the third number by the quotient. But, again, the process is simplified by dividing the product of the two extreme numbers by the middle number. This square is also 'associated' by multiplication. It is derived from the multiplying square by merely reversing the diagonals, and the constant of the multiplying square is the cube of that of the dividing square derived from it.

The next set of diagrams shows the solutions for the fifth order of square. They are all 'associated' in the same way as before. The subtracting square is derived from the adding square by reversing the diagonals and exchanging opposite numbers in the centres of the borders, and the constant of one is again n times that of the other. The dividing square is derived from the multiplying square

ADDING

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

SUBTRACTING

9	24	25	8	11
23	21	7	12	16
22	6	13	20	4
10	14	19	5	3
15	18	1	2	17

MULTIPLYING

54	648	1	12	144
324	16	6	72	27
8	3	36	432	162
48	18	216	81	4
9	108	1296	2	24

DIVIDING

24	648	1296	12	9
324	81	6	18	27
162	3	36	432	8
48	72	216	16	4
144	108	1	2	54

in the same way, and the constant of the latter is the 5th power (that is the *n*th) of that of the former.

These squares are thus quite easy for odd orders. But the reader will probably find some difficulty over the even orders, concerning which I will leave him to make his own researches, merely propounding two little problems.

117. TWO NEW MAGIC SQUARES.

Construct a subtracting magic square with the first sixteen whole numbers that shall be 'associated' by subtraction. The constant is, of course, obtained by subtracting the first number from the second in line, the result from the third, and the result again from the fourth. Also construct a dividing magic square of the same order that shall be 'associated' by division. The constant is obtained by dividing the second number in a line by the first, the third by the quotient, and the fourth by the next quotient.

118. MAGIC SQUARES OF TWO DEGREES.

While reading a French mathematical work I happened to come across, the following statement: 'A very remarkable magic square of 8, in two degrees, has been constructed by M. Pfeffermann. In other words, he has managed to dispose the sixty-four first numbers on the squares of a chessboard in such a way that the sum of the numbers in every line, every column, and in each of the two diagonals, shall be the same; and more, that if one substitutes for all the numbers their squares, the square still remains magic.' I at once set to work to solve this problem, and, although it proved a very hard nut, one was rewarded by the discovery of some curious and beautiful laws that govern it. The reader may like to try his hand at the puzzle.

UNCLASSIFIED PROBLEMS

"A snapper up of unconsidered trifles."
Winter's Tale, iv. 2.

119. WHO WAS FIRST?

Anderson, Biggs, and Carpenter were staying together at a place by the seaside. One day they went out in a boat and were a mile at sea when a rifle was fired on shore in their direction. Why or by whom the shot was fired fortunately does not concern us, as no information on these points is obtainable, but from the facts I picked up we can get material for a curious little puzzle for the novice.

It seems that Anderson only heard the report of the gun, Biggs only saw the smoke, and Carpenter merely saw the bullet strike the water near them. Now, the question arises: Which of them first knew of the discharge of the rifle?

120. A WONDERFUL VILLAGE.

There is a certain village in Japan, situated in a very low valley, and yet the sun is nearer to the inhabitants every noon, by 3,000 miles and upwards, than when he either rises or sets to these people. In what part of the country is the village situated?

121. A CALENDAR PUZZLE.

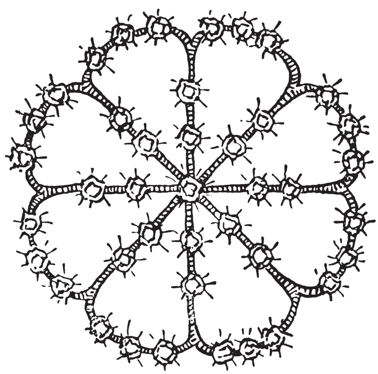
If the end of the world should come on the first day of a new century, can you say what are the chances that it will happen on a Sunday?

122. THE RUBY BROOCH.

The annals of Scotland Yard contain some remarkable cases of jewel robberies, but one of the most perplexing was the theft of Lady Littlewood's rubies. There have, of course, been many greater robberies in point of value, but few so artfully conceived. Lady Littlewood, of Romley Manor, had a beautiful but rather eccentric heirloom in the form of a ruby brooch. While staying at her town house early in the eighties she took the jewel to a shop in Brompton for some slight repairs.

"A fine collection of rubies, madam," said the shopkeeper, to whom her ladyship was a stranger.

"Yes," she replied; "but curiously enough I have never actually counted them. My mother once pointed out to me that if you start from the centre and count up one line, along the outside and down the next line, there are always eight rubies. So I should always know if a stone were missing."



Six months later a brother of Lady Littlewood's, who had returned from his regiment in India, noticed that his sister was wearing the ruby brooch one night at a county ball, and on their return home asked to look at it more closely. He immediately detected the fact that four of the stones were gone.

"How can that possibly be?" said Lady Littlewood. "If you count up one line from the centre, along the edge, and down the next line, in any direction, there are always eight stones. This was always so and is so now. How, therefore, would it be possible to remove a stone without my detecting it?"

"Nothing could be simpler," replied the brother. "I know the brooch well. It originally contained forty-five stones, and there are now only forty-one. Somebody has stolen four rubies, and then reset as small a number of the others as possible in such a way that there shall always be eight in any of the directions you have mentioned."

There was not the slightest doubt that the Brompton jeweller was the thief, and the matter was placed in the hands of the police. But the man was wanted for other robberies, and had left the neighbourhood some time before. To this day he has never been found.

The interesting little point that at first baffled the police, and which forms the subject of our puzzle, is this: How were the forty-five rubies originally arranged on the brooch? The illustration shows exactly how the forty-one were arranged after it came back from the jeweller; but although they count eight correctly in any of the directions mentioned, there are four stones missing.

123. PHEASANT-SHOOTING.

A Cockney friend, who is very apt to draw the long bow, and is evidently less of a sportsman than he pretends to be, relates to me the following not very credible yarn:—

"I've just been pheasant-shooting with my friend the duke. We had splendid sport, and I made some wonderful shots. What do you think of this, for instance? Perhaps you can twist it into a puzzle. The duke and I were crossing a field when suddenly twenty-four pheasants rose on the wing right in front of us. I fired, and two-thirds of them dropped dead at my feet. Then the duke had a shot at what were left, and brought down three-twenty-fourths of them, wounded in the wing. Now, out of those twenty-four birds, how many still remained?"

It seems a simple enough question, but can the reader give a correct answer?

SOLUTIONS

SOLUTIONS

1. AT A CATTLE MARKET.

Jakes must have taken 7 animals to market, Hodge must have taken 11, and Durrant must have taken 21. There were thus 39 animals altogether.

2. THE TWO AEROPLANES.

The man must have paid £500 and £750 for the two machines, making together £1,250; but as he sold them for only £1,200, he lost £50 by the transaction.

3. THE MILLIONAIRE'S PERPLEXITY.

The answer to this quite easy puzzle may, of course, be readily obtained by trial, deducting the largest power of 7 that is contained in one million dollars, then the next largest power from the remainder, and so on. But the little problem is intended to illustrate a simple direct method. The answer is given at once by converting 1,000,000 to the septenary scale, and it is on this subject of scales of notation that I propose to write a few words for the benefit of those who have never sufficiently considered the matter.

Our manner of figuring is a sort of perfected arithmetical shorthand, a system devised to enable us to manipulate numbers as rapidly and correctly as possible by means of symbols. If we write the number 2,341 to represent two thousand three hundred and forty-one dollars, we wish to imply 1 dollar, added to four times 10 dollars, added to three times 100 dollars, added to two times 1,000 dollars. From the number in the units place on the right, every figure to the left is understood to represent a multiple of the particular power of 10 that its position indicates, while a cipher (0) must be inserted where necessary in order to prevent confusion, for if instead of 207 we wrote 27 it would be obviously misleading. We thus only require ten figures, because directly a number exceeds 9 we put a second figure to the left, directly it exceeds 99 we put a third figure to the left, and so on. It will be seen that this is a purely arbitrary method. It is working in the denary (or ten) scale of notation, a system undoubtedly derived from the fact that our forefathers who devised it had ten fingers upon which they were accustomed to count, like our children of to-day. It is unnecessary for us ordinarily to state that we are using the denary scale, because this is always understood in the common affairs of life.

But if a man said that he had 6,553 dollars in the septenary (or seven) scale of notation, you will find that this is precisely the same amount as 2,341 in our ordinary denary scale. Instead of using powers of ten, he uses powers of 7, so that he never needs any figure higher than 6, and 6,553 really stands for 3, added to five times 7, added to five times 49, added to six times 343 (in the ordinary notation), or 2,341. To reverse the operation, and convert 2,341 from the denary to the septenary scale, we divide it by 7, and get 334 and remainder 3; divide 334 by 7, and get 47 and remainder 5; and so keep on dividing by 7 as long as there is anything to divide. The remainders, read backwards, 6, 5, 5, 3, give us the answer, 6,553.

Now, as I have said, our puzzle may be solved at once by merely converting

1,000,000 dollars to the septenary scale. Keep on dividing this number by 7 until there is nothing more left to divide, and the remainders will be found to be 11333311 which is 1,000,000 expressed in the septenary scale. Therefore, 1 gift of 1 dollar, 1 gift of 7 dollars, 3 gifts of 49 dollars, 3 gifts of 343 dollars, 3 gifts of 2,401 dollars, 3 gifts of 16,807 dollars, 1 gift of 117,649 dollars, and one substantial gift of 823,543 dollars, satisfactorily solves our problem. And it is the only possible solution. It is thus seen that no "trials" are necessary; by converting to the septenary scale of notation we go direct to the answer.

4. THE GROCER AND DRAPER.

The grocer was delayed half a minute and the draper eight minutes and a half (seventeen times as long as the grocer), making together nine minutes. Now, the grocer took twenty-four minutes to weigh out the sugar, and, with the half-minute delay, spent 24 min. 30 sec. over the task; but the draper had only to make *forty-seven* cuts to divide the roll of cloth, containing forty-eight yards, into yard pieces! This took him 15 min. 40 sec., and when we add the eight minutes and a half delay we get 24 min. 10 sec., from which it is clear that the draper won the race by twenty seconds. The majority of solvers make forty-eight cuts to divide the roll into forty-eight pieces!

5. JUDKINS'S CATTLE.

As there were five droves with an equal number of animals in each drove, the number must be divisible by 5; and as every one of the eight dealers bought the same number of animals, the number must be divisible by 8. Therefore the number must be a multiple of 40. The highest possible multiple of 40 that will work will be found to be 120, and this number could be made up in one of two ways–1 ox, 23 pigs, and 96 sheep, or 3 oxen, 8 pigs, and 109 sheep. But the first is excluded by the statement that the animals consisted of "oxen, pigs, and sheep," because a single ox is not oxen. Therefore the second grouping is the correct answer.

6. MAMMA'S AGE.

The age of Mamma must have been 29 years 2 months; that of Papa, 35 years; and that of the child, Tommy, 5 years 10 months. Added together, these make seventy years. The father is six times the age of the son, and, after 23 years 4 months have elapsed, their united ages will amount to 140 years, and Tommy will be just half the age of his father.

7. THEIR AGES.

The gentleman's age must have been 54 years and that of his wife 45 years.

8. ROVER'S AGE.

Rover's present age is ten years and Mildred's thirty years. Five years ago their respective ages were five and twenty-five. Remember that we said "four times older than the dog," which is the same as "five times as old."

9. CONCERNING TOMMY'S AGE.

Tommy Smart's age must have been nine years and three-fifths. Ann's age was sixteen and four-fifths, the mother's thirty-eight and two-fifths, and the father's fifty and two-fifths.

10. THE BAG OF NUTS.

It will be found that when Herbert takes twelve, Robert and Christopher will take nine and fourteen respectively, and that they will have together taken thirty-five nuts. As 35 is contained in 770 twenty-two times, we have merely to multiply 12, 9, and 14 by 22 to discover that Herbert's share was 264, Robert's 198, and Christopher's 308. Then, as the total of their ages is 17½ years or half the sum of 12, 9, and 14, their respective ages must be 6, 4½, and 7 years.

11. A FAMILY PARTY.

The party consisted of two little girls and a boy, their father and mother, and their father's father and mother.

12. A TIME PUZZLE.

Twenty-six minutes.

13. THE WAPSHAW'S WHARF MYSTERY.

There are eleven different times in twelve hours when the hour and minute hands of a clock are exactly one above the other. If we divide 12 hours by 11 we get 1 hr. 5 min. 27$\frac{3}{11}$ sec., and this is the time after twelve o'clock when they are first together, and also the time that elapses between one occasion of the hands being together and the next. They are together for the second time at 2 hr. 10 min. 54 $\frac{3}{11}$ sec. (twice the above time); next at 3 hr. 16 min. 21 $\frac{9}{11}$ sec.; next at 4 hr. 21 min. 49 $\frac{1}{11}$ sec. This last is the only occasion on which the two hands are together with the second hand "just past the forty-ninth second." This, then, is the time at which the watch must have stopped. Guy Boothby, in the opening sentence of his *Across the World for a Wife*, says, "It was a cold, dreary winter's afternoon, and by the time the hands of the clock on my mantelpiece joined forces and stood at twenty minutes past four, my chambers were well-nigh as dark as midnight." It is evident that the author he is 1 min. 49 $\frac{1}{11}$ sec. out in his reckoning.

14. CHANGING PLACES.

There are thirty-six pairs of times when the hands exactly change places between three p.m. and midnight. The number of pairs of times from any hour (n) to midnight is the sum of 12 - (n + 1) natural numbers. In the case of the puzzle $n = 3$; therefore 12 - (3 + 1) = 8 and 1 + 2 + 3 4 + 5 + 6 + 7 + 8 = 36, the required answer.

The first pair of times is 3 hr. 21 $\frac{57}{143}$ min. and 4 hr. 16$\frac{112}{143}$min., and the last pair is 10 hr. 59 $\frac{83}{143}$ min. and 11 hr. 54 $\frac{138}{143}$ min. I will not give all the remainder of the thirty-six pairs of times, but supply a formula by which any of the sixty-six pairs that occur from midday to midnight may be at once found: —

$$a \text{ hr } \frac{720b+60a}{143} \text{ min. and } b \text{ hr. } \frac{720a+60b}{143} \text{ min.}$$

For the letter a may be substituted any hour from 0, 1, 2, 3 up to 10 (where nought stands for 12 o'clock midday); and b may represent any hour, later than a, up to 11.

By the aid of this formula there is no difficulty in discovering the answer to the second question: $a = 8$ and $b = 11$ will give the pair 8 hr. 58 $\frac{106}{143}$ min. and 11 hr. 44 $\frac{128}{143}$ min., the latter being the time when the minute hand is nearest of all to the point IX—in fact, it is only $\frac{15}{143}$ of a minute distant.

Readers may find it instructive to make a table of all the sixty-six pairs of times when the hands of a clock change places. An easy way is as follows: Make a column for the first times and a second column for the second times of the pairs. By making $a = 0$ and $b = 1$ in the above expressions we find the first case, and enter hr. $5\frac{5}{143}$ min. at the head of the first column, and 1 hr. $\frac{60}{143}$ min. at the head of the second column. Now, by successively adding $5\frac{5}{143}$ min. in the first, and 1 hr. $\frac{60}{143}$ min. in the second column, we get all the *eleven* pairs in which the first time is a certain number of minutes after nought, or mid-day. Then there is a "jump" in the times, but you can find the next pair by making $a = 1$ and $b = 2$, and then by successively adding these two times as before you will get all the *ten* pairs after 1 o'clock. Then there is another "jump," and you will be able to get by addition all the *nine* pairs after 2 o'clock. And so on to the end. I will leave readers to investigate for themselves the nature and cause of the "jumps." In this way we get under the successive hours, 11 + 10 + 9 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 66 pairs of times, which result agrees with the formula in the first paragraph of this article.

Some time ago the principal of a Civil Service Training College, who conducts a "Civil Service Column" in one of the periodicals, had the query addressed to him, "How soon after XII o'clock will a clock with both hands of the same length be ambiguous?" His first answer was, "Some time past one o'clock," but he varied the answer from issue to issue. At length some of his readers convinced him that the answer is, "At $5\frac{5}{143}$ min. past XII;" and this he finally gave as correct, together with the reason for it that at that time th*e time indicated is the same whichever hand you may assume as hour hand!*

15. THE STOP-WATCH.

The time indicated on the watch was $5\frac{5}{11}$ min. past 9, when the second hand would be at $27\frac{3}{11}$ sec. The next time the hands would be similar distances apart would be $54\frac{6}{11}$ min. past 2, when the second hand would be at $32\frac{5}{11}$ sec. But you need only hold the watch (or our previous illustration of it) in front of a mirror, when you will see the second time reflected in it! Of course, when reflected, you will read XI as I, X as II, and so on.

16. THE THREE CLOCKS.

As a mere arithmetical problem this question presents no difficulty. In order that the hands shall all point to twelve o'clock at the same time, it is necessary that B shall gain at least twelve hours and that C shall lose twelve hours. As B gains a minute in a day of twenty-four hours, and C loses a minute in precisely the same time, it is evident that one will have gained 720 minutes (just twelve hours) in 720 days, and the other will have lost 720 minutes in 720 days. Clock A keeping perfect time, all three clocks must indicate twelve o'clock simultaneously at noon on the 720th day from April 1, 1898. What day of the month will that be?

I published this little puzzle in 1898 to see how many people were aware of the fact that 1900 would not be a leap year. It was surprising how many were then ignorant on the point. Every year that can be divided by four without a remainder is bissextile or leap year, with the exception that one leap year is cut off in the century. 1800 was not a leap year, nor was 1900. On the other hand, however, to make the calendar more nearly agree with the sun's course, every fourth hundred year is still considered bissextile. Consequently, 2000, 2400, 2800, 3200, etc., will all be leap years. May my readers live to see them. We therefore find that 720 days from noon of April 1, 1898, brings us to noon of March 22, 1900.

17. THE VILLAGE SIMPLETON.

The day of the week on which the conversation took place was Sunday. For when the day after to-morrow (Tuesday) is "yesterday," "to-day" will be Wednesday; and when the day before yesterday (Friday) was "to-morrow," "to-day" was Thursday. There are two days between Thursday and Sunday, and between Sunday and Wednesday.

18. THE TWO TRAINS.

One train was running just twice as fast as the other.

19. THE THREE VILLAGES.

Calling the three villages by their initial letters, it is clear that the three roads form a triangle, A, B, C, with a perpendicular, measuring twelve miles, dropped from C to the base A, B. This divides our triangle into two right-angled triangles with a twelve-mile side in common. It is then found that the distance from A to C is 15 miles, from C to B 20 miles, and from A to B 25 (that is 9 and 16) miles. These figures are easily proved, for the square of 12 added to the square of 9 equals the square of 15, and the square of 12 added to the square of 16 equals the square of 20.

20. DONKEY RIDING.

The complete mile was run in nine minutes. From the facts stated we cannot determine the time taken over the first and second quarter-miles separately, but together they, of course, took four and a half minutes. The last two quarters were run in two and a quarter minutes each.

21. THE BARREL OF BEER.

Here the digital roots of the six numbers are 6, 4, 1, 2, 7, 9, which together sum to 29, whose digital root is 2. As the contents of the barrels sold must be a number divisible by 3, if one buyer purchased twice as much as the other, we must find a barrel with root 2, 5, or 8 to set on one side. There is only one barrel, that containing 20 gallons, that fulfils these conditions. So the man must have kept these 20 gallons of beer for his own use and sold one man 33 gallons (the 18-gallon and 15-gallon barrels) and sold the other man 66 gallons (the 16, 19, and 31 gallon barrels).

22. THE LOCKERS PUZZLE.

The smallest possible total is 356 = 107 + 249, and the largest sum possible is 981 = 235 + 746, or 657+324. The middle sum may be either 720 = 134 + 586, or 702 = 134 + 568, or 407 = 138 + 269. The total in this case must be made up of three of the figures 0, 2, 4, 7, but no sum other than the three given can possibly be obtained. We have therefore no choice in the case of the first locker, an alternative in the case of the third, and any one of three arrangements in the case of the middle locker. Here is one solution:—

107	134	235
249	586	746
356	720	981

Of course, in each case figures in the first two lines may be exchanged vertically without altering the total, and as a result there are just 3,072 different ways in which the figures might be actually placed on the locker doors. I must content myself with showing one little principle involved in this puzzle. The sum of the digits in the total is always governed by the digit omitted. $\frac{9}{9} - \frac{7}{10} - \frac{5}{11} - \frac{3}{12} - \frac{1}{13} - \frac{8}{14} - \frac{6}{15} - \frac{4}{16} - \frac{2}{17} - \frac{0}{18}$. Whichever digit shown here in the upper line we omit, the sum of the digits in the total will be found beneath it. Thus in the case of locker A we omitted 8, and the figures in the total sum up to 14. If, therefore, we wanted to get 356, we may know at once to a certainty that it can only be obtained (if at all) by dropping the 8.

23. THE THREE GROUPS.

There are nine solutions to this puzzle, as follows, and no more:—

12 × 483 = 5,796 27 × 198 = 5,346
42 × 138 = 5,796 39 × 186 = 7,254
18 × 297 = 5,346 48 × 159 = 7,632
28 × 157 = 4,396
4 × 1,738 = 6,952
4 × 1,963 = 7,852

The seventh answer is the one that is most likely to be overlooked by solvers of the puzzle.

24. THE PIERROT'S PUZZLE.

There are just six different solutions to this puzzle, as follows:—

8	multiplied by	473	equals	3784
9	"	351	"	3159
15	"	93	"	1395
21	"	87	"	1287
27	"	81	"	2187
35	"	41	"	1435

It will be seen that in every case the two multipliers contain exactly the same figures as the product.

25. THE CAB NUMBERS.

The highest product is, I think, obtained by multiplying 8,745,231 by 96–namely, 839,542,176.

Dealing here with the problem generally, I have shown in the last puzzle that with three digits there are only two possible solutions, and with four digits only six different solutions.

These cases have all been given. With five digits there are just twenty-two solutions, as follows:—

3	×	4128	=	12384
3	×	4281	=	12843
3	×	7125	=	21375
3	×	7251	=	21753
2541	×	6	=	15246
651	×	24	=	15624
678	×	42	=	28476
246	×	51	=	12546
57	×	834	=	47538
75	×	231	=	17325
624	×	78	=	48672
435	×	87	=	37845

9	×	7461	=	67149
72	×	936	=	67392

2	×	8714	=	17428
2	×	8741	=	17482
65	×	281	=	18265
65	×	983	=	63985

4973	×	8	=	39784
6521	×	8	=	52168
14	×	926	=	12964
86	×	251	=	21586

Now, if we took every possible combination and tested it by multiplication, we should need to make no fewer than 30,240 trials, or, if we at once rejected the number 1 as a multiplier, 28,560 trials—a task that I think most people would be inclined to shirk. But let us consider whether there be no shorter way of getting at the results required. I have already explained that if you add together the digits of any number and then, as often as necessary, add the digits of the result, you must ultimately get a number composed of one figure. This last number I call the "digital root." It is necessary in every solution of our problem that the root of the sum of the digital roots of our multipliers shall be the same as the root of their product. There are only four ways in which this can happen: when the digital roots of the multipliers are 3 and 6, or 9 and 9, or 2 and 2, or 5 and 8. I have divided the twenty-two answers above into these four classes. It is thus evident that the digital root of any product in the first two classes must be 9, and in the second two classes 4.

Owing to the fact that no number of five figures can have a digital sum less than 15 or more than 35, we find that the figures of our product must sum to either 18 or 27 to produce the root 9, and to either 22 or 31 to produce the root 4. There are 3 ways of selecting five different figures that add up to 18, there are 11 ways of selecting five figures that add up to 27, there are 9 ways of selecting five figures that add up to 22, and 5 ways of selecting five figures that add up to 31. There are, therefore, 28 different groups, and no more, from any one of which a product may be formed.

We next write out in a column these 28 sets of five figures, and proceed to tabulate the possible factors, or multipliers, into which they may be split. Roughly speaking, there would now appear to be about 2,000 possible cases to be tried, instead of the 30,240 mentioned above; but the process of elimination now begins, and if the reader has a quick eye and a clear head he can rapidly dispose of the large bulk of these cases, and there will be comparatively few test multiplications necessary. It would take far too much space to explain my own method in detail, but I will take the first set of figures in my table and show how easily it is done by the aid of little tricks and dodges that should occur to everybody as he goes along.

My first product group of five figures is 84,321. Here, as we have seen, the root of each factor must be 3 or a multiple of 3. As there is no 6 or 9, the only single multiplier is 3. Now, the remaining four figures can be arranged in 24 different ways, but there is no need to make 24 multiplications. We see at a glance that, in order to get a five-figure product, either the 8 or the 4 must be the first figure to the left. But unless the 2 is preceded on the right by the 8, it will produce when multiplied either a 6 or a 7, which must not occur. We are, therefore, reduced at once to the two cases, 3 × 4,128 and 3 x 4,281, both of which give correct solutions. Suppose next that we are trying the two-figure factor, 21. Here we see that if the number to be multiplied is under 500 the product will either have only four figures or begin with 10. Therefore we have only to examine the cases 21 × 843 and 21 × 834. But we know that the first figure will be repeated, and that the second figure will be twice the first figure added to the second. Consequently, as twice 3 added to 4 produces a nought in our product, the first case is at once rejected. It only remains to try the remaining case by multiplication, when we find it does not give a correct answer. If we are next trying the factor 12, we see at the start that neither the 8 nor the 3 can be in the units place, because they would produce a 6, and so on. A sharp eye and an alert judgement will enable us thus to run through our table in a much shorter time than would be expected. The process took me a little more than three hours.

I have not attempted to enumerate the solutions in the cases of six, seven, eight, and nine digits, but I have recorded nearly fifty examples with nine digits alone.

26. THE FOUR SEVENS.

The way to write four sevens with simple arithmetical signs so that they represent 100 is as follows:—

$$\frac{7}{.7} \times \frac{7}{.7} = 100.$$

Of course the fraction, 7 over decimal 7, equals 7 divided by $\frac{7}{10}$, which is the same as 70 divided by 7, or 10. Then 10 multiplied by 10 is 100, and there you are! It will be seen that this solution applies equally to any number whatever that you may substitute for 7.

27. THE THIRTY-THREE PEARLS.

The value of the large central pearl must have been £3,000. The pearl at one end (from which they increased in value by £100) was £1,400; the pearl at the other end, £600.

28. PAINTING THE LAMP-POSTS.

Pat must have painted six more posts than Tim, no matter how many lamp-posts there were. For example, suppose twelve on each side; then Pat painted fifteen and Tim nine. If a hundred on each side, Pat painted one hundred and three, and Tim only ninety-seven

29. THE TORN NUMBER.

The other number that answers all the requirements of the puzzle is 9,801. If we divide this in the middle into two numbers and add them together we get 99, which, multiplied by itself, produces 9,801. It is true that 2,025 may be treated in the same way, only this number is excluded by the condition which requires that no two figures should be alike.

The general solution is curious. Call the number of figures in each half of the torn label n. Then, if we add 1 to each of the exponents of the prime factors (other than 3) of $10n - 1$ (1 being regarded as a factor with the constant exponent, 1), their product will be the number of solutions. Thus, for a label of six figures, $n = 3$. The factors of $10^3 - 1$ are $1^1 \times 37^1$ (not considering the 3^3), and the product of $2 \times 2 = 4$, the number of solutions. This always includes the special cases 98 - 01, 00 - 01, 998 - 01, 000 - 001, etc. The solutions are obtained as follows:—Factorize $10^3 - 1$ in all possible ways, always keeping the powers of 3 together, thus, 37×27, 999×1. Then solve the equation $37x = 27y + 1$. Here $x = 19$ and $y = 26$. Therefore, $19 \times 37 = 703$, the square of which gives one label, 494,209. A complementary solution (through $27x = 37x + 1$) can at once be found by $10^n - 703 = 297$, the square of which gives 088,209 for second label. (These non-significant noughts to the left must be included, though they lead to peculiar cases like $00238 - 04641 = 4879^2$, where 0238 - 4641 would not work.) The special case 999×1 we can write at once 998,001, according to the law shown above, by adding nines on one half and noughts on the other, and its complementary will be 1 preceded by five noughts, or 000001. Thus we get the squares of 999 and 1. These are the four solutions.

30.CIRCLING THE SQUARES.

Though this problem might strike the novice as being rather difficult, it is, as a matter of fact, quite easy, and is made still easier by inserting four out of the ten numbers.

First, it will be found that squares that are diametrically opposite have a common difference. For example, the difference between the square of 14 and the square of 2, in the diagram, is 192; and the difference between the square of 16 and the square of 8 is also 192. This must be so in every case. Then it should be remembered that the difference between squares of two consecutive numbers is always twice the smaller number plus 1, and that the difference between the squares of any two numbers can always be expressed as the difference of the numbers multiplied by their sum. Thus the square of 5 (25) less the square of 4 (16) equals (2 × 4) + 1, or 9; also, the square of 7 (49) less the square of 3 (9) equals (7 + 3) × (7 - 3), or 40.

Now, the number 192, referred to above, may be divided into five different pairs of even factors: 2 × 96, 4 × 48, 6 × 32, 8 × 24, and 12 × 16, and these divided by 2 give us, 1 × 48, 2 × 24, 3 × 16, 4 × 12, and 6 × 8. The difference and sum respectively of each of these pairs in turn produce 47, 49; 22, 26; 13, 19; 8, 16; and 2, 14. These are the required numbers, four of which are already placed. The six numbers that have to be added may be placed in just six different ways, one of which is as follows, reading round the circle clockwise: 16, 2, 49, 22, 19, 8, 14, 47, 26, 13.

I will just draw the reader's attention to one other little point. In all circles of this kind, the difference between diametrically opposite numbers increases by a certain ratio, the first numbers (with the exception of a circle of 6) being 4 and 6, and the others formed by doubling the next preceding but one. Thus, in the above case, the first difference is 2, and then the numbers increase by 4, 6, 8, and 12. Of course, an infinite number of solutions may be found if we admit fractions. The number of squares in a circle of this kind must, however, be of the form $4n + 6$; that is, it must be a number composed of 6 plus a multiple of 4.

31. THE FARMER AND HIS SHEEP.

The farmer had one sheep only! If he divided this sheep (which is best done by weight) into two parts, making one part two-thirds and the other part one-third, then the difference between these two numbers is the same as the difference between their squares—that is, one-third. Any two fractions will do if the denominator equals the sum of the two numerators.

32. THE ARTILLERYMEN'S DILEMMA.

We were required to find the smallest number of cannon balls that we could lay on the ground to form a perfect square, and could pile into a square pyramid. I will try to make the matter clear to the merest novice.

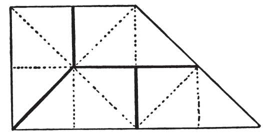
1	2	3	4	5	6	7
1	3	6	10	15	21	28
1	4	10	20	35	56	84
1	5	14	30	55	91	140

Here in the first row we place in regular order the natural numbers. Each number in the second row represents the sum of the numbers in the row above, from the beginning to the number just over it. Thus 1, 2, 3, 4, added together, make 10. The third row is formed in exactly the same way as the second. In the fourth row every number is formed by adding together the number just above it and the preceding number. Thus 4 and 10 make 14, 20 and 35 make 55. Now, all the numbers in the second row are triangular numbers, which means that these numbers of cannon balls may be laid out on the ground so as to form equilateral triangles. The numbers in the third row will all form our triangular pyramids, while the numbers in the fourth row will all form square pyramids.

Thus the very process of forming the above numbers shows us that every square pyramid is the sum of two triangular pyramids, one of which has the same number of balls in the side at the base, and the other one ball fewer. If we continue the above table to twenty-four places, we shall reach the number 4,900 in the fourth row. As this number is the square of 70, we can lay out the balls in a square, and can form a square pyramid with them. This manner of writing out the series until we come to a square number does not appeal to the mathematical mind, but it serves to show how the answer to the particular puzzle may be easily arrived at by anybody. As a matter of fact, I confess my failure to discover any number other than 4,900 that fulfils the conditions, nor have I found any rigid proof that this is the only answer. The problem is a difficult one, and the second answer, if it exists (which I do not believe), certainly runs into big figures.

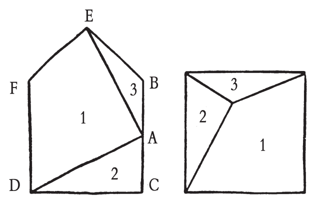
For the benefit of more advanced mathematicians I will add that the general expression for square pyramid numbers is $\frac{2n^3 + 3n^2 + n}{6}$. For this expression to be also a square number (the special case of 1 excepted) it is necessary that $n = p^2 - 1 = 6t^2$, where $2p^2 - 1 = q^2$ (the "Pellian Equation"). In the case of our solution above, $n = 24$, $p = 5$, $t = 2$, $q = 7$.

33. AN EASY DISSECTION PUZZLE.



The solution to this puzzle is shown in the illustration. Divide the figure up into twelve equal triangles, and it is easy to discover the directions of the cuts, as indicated by the dark lines.

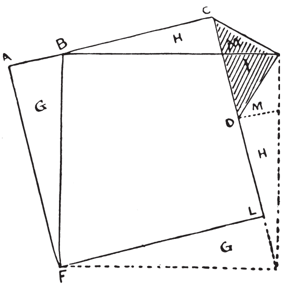
34. THE JOINER'S PROBLEM.



Nothing could be easier than the solution of this puzzle—when you know how to do it. And yet it is apt to perplex the novice a good deal if he wants to do it in the fewest possible pieces—three. All you have to do is to find the point A, midway between B and C, and then cut from A to D and from A to E. The three pieces then form a square in the manner shown. Of course, the proportions of the original figure must be correct; thus the triangle BEF is just a quarter of the square BCDF. Draw lines from B to D and from C to F and this will be clear.

35. ANOTHER JOINER'S PROBLEM.

The point was to find a general rule for forming a perfect square out of another square combined with a "right-angled isosceles triangle." The triangle to which geometricians give this high-sounding name is, of course, nothing more or less than half a square that has been divided from corner to corner.



The precise relative proportions of the square and triangle are of no consequence whatever. It is only necessary to cut the wood or material into five pieces.

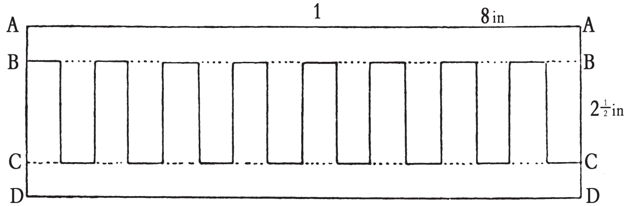
Suppose our original square to be ACLF in the above diagram and our triangle to be the shaded portion CED. Now, we first find half the length of the long side of the triangle (CD) and measure off this length at AB. Then we place the triangle in its present position against the square and make two cuts—one from B to F, and the other from B to E. Strange as it may seem, that is all that is necessary! If we now remove the pieces G, H, and M to their new places, as shown in the diagram, we get the perfect square BEKF.

Take any two square pieces of paper, of different sizes but perfect squares, and cut the smaller one in half from corner to corner. Now proceed in the manner shown, and you will find that the two pieces may be combined to form a larger square by making these two simple cuts, and that no piece will be required to be turned over.

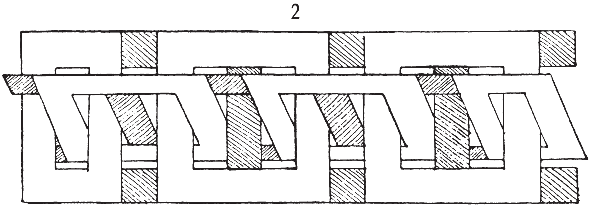
The remark that the triangle might be "a little larger or a good deal smaller in proportion" was intended to bar cases where area of triangle is greater than area of square. In such cases six pieces are necessary, and if triangle and square are of equal area there is an obvious solution in three pieces, by simply cutting the square in half diagonally.

36. THE CARDBOARD CHAIN.

1



2

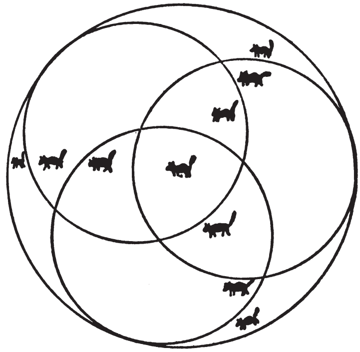


The reader will probably feel rewarded for any care and patience that he may bestow on cutting out the cardboard chain. We will suppose that he has a piece of cardboard measuring 8 in. by 2½ in., though the dimensions are of no importance. Yet if you want a long chain you must, of course, take a long strip of cardboard. First rule pencil lines B B and C C, half an inch from the edges, and also the short perpendicular lines half an inch apart. (See next page.) Rule lines on the other side in just the same way, and in order that they shall coincide it is well to prick through the card with a needle the points where the short lines end. Now take your penknife and split the card from A A down to B B, and from D D up to C C. Then cut right through the card along all the short perpendicular lines, and half through the card along the short portions of B B and C C that are not dotted. Next turn the card over and cut half through along the short lines on B B and C C at the places that are immediately beneath the dotted lines on the upper side. With a little careful separation of the parts with the penknife, the cardboard may now be divided into two interlacing ladder-like portions, as shown in Fig. 2; and if you cut away all the shaded parts you will get the chain, cut solidly out of the cardboard, without any join, as shown in the illustrations on page 40.

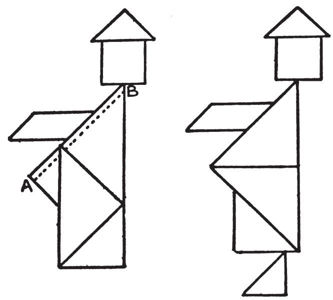
It is an interesting variant of the puzzle to cut out two keys on a ring—in the same manner without join.

37. THE WIZARD'S CATS.

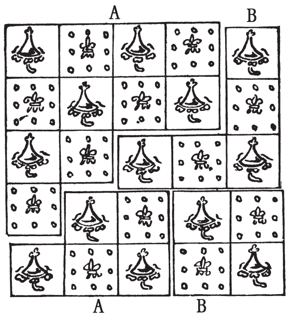
The illustration requires no explanation. It shows clearly how the three circles may be drawn so that every cat has a separate enclosure, and cannot approach another cat without crossing a line.



38. A TANGRAM PARADOX.



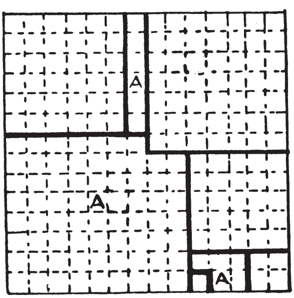
The diagrams will show how the figures are constructed—each with the seven Tangrams. It will be noticed that in both cases the head, hat, and arm are precisely alike, and the width at the base of the body the same. But this body contains four pieces in the first case, and in the second design only three. The first is larger than the second by exactly that narrow strip indicated by the dotted line between A and B. This strip is therefore exactly equal in area to the piece forming the foot in the other design, though when thus distributed along the side of the body the increased dimension is not easily apparent to the eye.

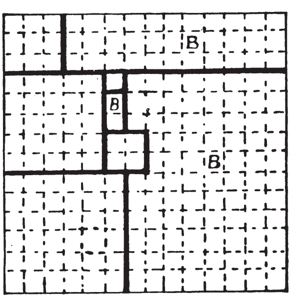


39. THE CUSHION COVERS.

The two pieces of brocade marked A will fit together and form one perfect square cushion top, and the two pieces marked B will form the other.

40. MRS. SMILEY'S CHRISTMAS PRESENT.



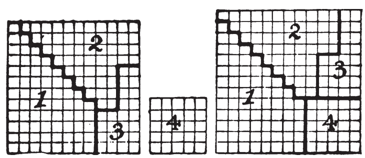


The first step is to find six different square numbers that sum to 196. For example,

$$1 + 4 + 25 + 36 + 49 + 81 = 196;$$
$$1 + 4 + 9 + 25 + 36 + 121 = 196;$$
$$1 + 9 + 16 + 25 + 64 + 81 = 196.$$

The rest calls for individual judgment and ingenuity, and no definite rules can be given for procedure. The annexed diagrams will show solutions for the first two cases stated. Of course the three pieces marked A and those marked B will fit together and form a square in each case. The assembling of the parts may be slightly varied, and the reader may be interested in finding a solution for the third set of squares I have given.

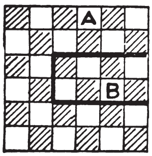
41. ANOTHER PATCHWORK PUZZLE.



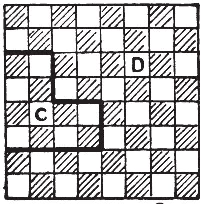
The lady need only unpick the stitches along the dark lines in the larger portion of patchwork, when the four pieces will fit together and form a square, as shown in our illustration.

42. LINOLEUM CUTTING.

There is only one solution that will enable us to retain the larger of the two pieces with as little as possible cut from it. Fig. 1 in the following diagram shows how the smaller piece is to be cut, and Fig. 2 how we should dissect the larger piece, while in Fig. 3 we have the new square 10 × 10 formed by the four pieces with all the chequers properly matched. It will be seen that the piece D contains fifty-two chequers, and this is the largest piece that it is possible to preserve under the conditions.

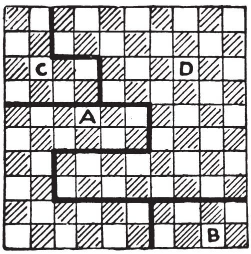


1

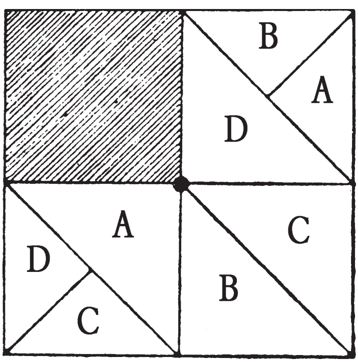


2

3



43. THE FOUR SONS.

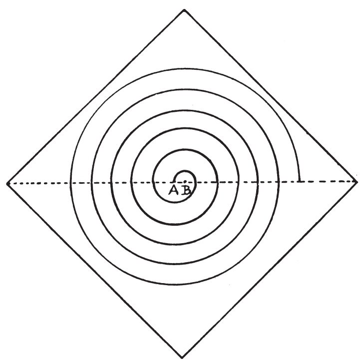


The diagram shows the most equitable division of the land possible, "so that each son shall receive land of exactly the same area and exactly similar in shape," and so that each shall have access to the well in the centre without trespass on another's land. The conditions do not require that each son's land shall be in one piece, but it is necessary that the two portions assigned to an individual should be kept apart, or two adjoining portions might be held to be one piece, in which case the condition as to shape would have to be broken. At present there is only one shape for each piece of land—half a square divided diagonally. And A, B, C, and D can each reach their land from the outside, and have each equal access to the well in the centre.

44. THE THREE RAILWAY STATIONS.

The three stations form a triangle, with sides 13, 14, and 15 miles. Make the 14 side the base; then the height of the triangle is 12 and the area 84. Multiply the three sides together and divide by four times the area. The result is eight miles and one-eighth, the distance required.

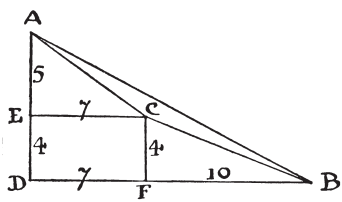
45. DRAWING A SPIRAL.



Make a fold in the paper, as shown by the dotted line in the illustration. Then, taking any two points, as A and B, describe semicircles on the line alternately from the centres B and A, being careful to make the ends join, and the thing is done. Of course this is not a *true* spiral, but the puzzle was to produce the *particular* spiral that was shown, and that was drawn in this simple manner.

46. THE YORKSHIRE ESTATES.

The triangular piece of land that was not for sale contains exactly eleven acres. Of course it is not difficult to find the answer if we follow the eccentric and tricky tracks of intricate trigonometry; or I might say that the application of a well-known formula reduces the problem to finding one-quarter of the square root of (4 × 370 × 116) - (370 + 116 - 74)^2—that is a quarter of the square root of 1936, which is one-quarter of 44, or 11 acres. But all that the reader really requires to know is the Pythagorean law on which many puzzles have been built, that in any right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. I shall dispense with all "surds" and similar absurdities, notwithstanding the fact that the sides of our triangle are clearly incommensurate, since we cannot exactly extract the square roots of the three square areas.



In the above diagram ABC represents our triangle. ADB is a right-angled triangle, AD measuring 9 and BD measuring 17, because the square of 9 added to the square of 17 equals 370, the known area of the square on AB. Also AEC is a right-angled triangle, and the square of 5 added to the square of 7 equals 74, the square estate on A C. Similarly, CFB is a right-angled triangle, for the square of 4 added to the square of 10 equals 116, the square estate on BC. Now, although the sides of our triangular estate are incommensurate, we have in this diagram all the exact figures that we need to discover the area with precision.

The area of our triangle ADB is clearly half of 9 × 17, or 76½ acres. The area of AEC is half of 5 × 7, or 17½ acres; the area of CFB is half of 4 × 10, or 20 acres; and the area of the oblong EDFC is obviously 4 × 7, or 28 acres. Now, if we add together 17½, 20, and 28 = 65½, and deduct this sum from the area of the large triangle ADB (which we have found to be 76½ acres), what remains must clearly be the area of ABC. That is to say, the area we want must be 76½ - 65½ = 11 acres exactly.

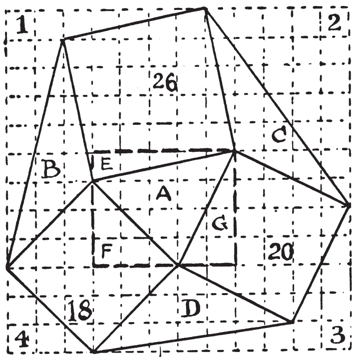
47. FARMER WURZEL'S ESTATE.

The area of the complete estate is exactly one hundred acres. To find this answer I use the following little formula,

$$\frac{\sqrt{4ab - (a + b + c)^2}}{4}$$

where *a*, *b*, *c* represent the three square areas, in any order. The expression gives the area of the triangle A. This will be found to be 9 acres. It can be easily proved that A, B, C, and D are all equal in area; so the answer is 26 + 20 + 18 + 9 + 9 + 9 + 9 = 100 acres.

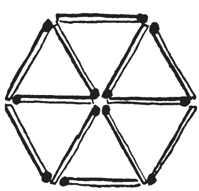
Here is the proof. If every little dotted square in the diagram represents an acre, this must be a correct plan of the estate, for the squares of 5 and 1 together equal 26; the squares of 4 and 2 equal 20; and the squares of 3 and 3 added together equal 18. Now we see at once that the area of the triangle E is 2½, F is 4½, and G is 4. These added together make 11 acres, which we deduct from the area of the rectangle, 20 acres, and we find that the field A contains exactly 9 acres. If you want to prove that B, C, and D are equal in size to A, divide them in two by a line from the middle of the longest side to the opposite angle, and you will find that the two pieces in every case, if cut out, will exactly fit together and form A.



Or we can get our proof in a still easier way. The complete area of the squared diagram is 12 × 12 = 144 acres, and the portions 1, 2, 3, 4, not included in the estate, have the respective areas of 12½, 17½, 9½, and 4½. These added together make 44, which, deducted from 144, leaves 100 as the required area of the complete estate.

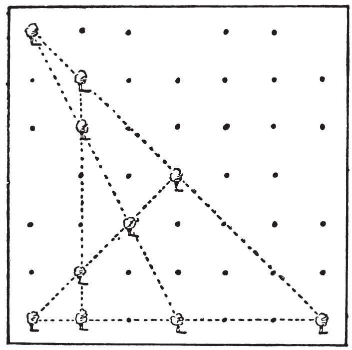
48. THE SIX SHEEP-PENS.

Place the twelve matches in the manner shown in the illustration, and you will have six pens of equal size.



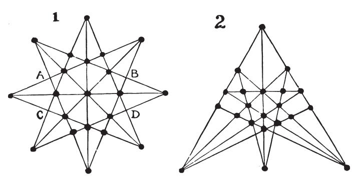
49. A PLANTATION PUZZLE.

The illustration shows the ten trees that must be left to form five rows with four trees in every row. The dots represent the positions of the trees that have been cut down.

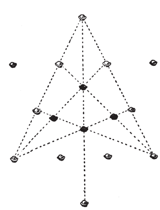


50. THE TWENTY-ONE TREES.

I give two pleasing arrangements of the trees. In each case there are twelve straight rows with five trees in every row.



51. THE TWELVE MINCE-PIES.



If you ignore the four black pies in our illustration, the remaining twelve are in their original positions. Now remove the four detached pies to the places occupied by the black ones, and you will have your seven straight rows of four, as shown by the dotted lines.

52. THE GRASSHOPPER PUZZLE.

Move the counters in the following order. The moves in brackets are to be made four times in succession. 12, 1, 3, 2, 12, 11, 1, 3, 2 (5, 7, 9, 10, 8, 6, 4), 3, 2, 12, 11, 2, 1, 2. The grasshoppers will then be reversed in forty-four moves.

The general solution of this problem is very difficult. Of course it can always be solved by the method given in the solution of the last puzzle, if we have no desire to use the fewest possible moves. But to employ a full economy of moves we have two main points to consider. There are always what I call a lower movement (L) and an upper movement (U). L consists in exchanging certain of the highest numbers, such as 12, 11, 10 in our "Grasshopper Puzzle," with certain of the lower numbers, 1, 2, 3; the former moving in a clockwise direction, the latter in a non-clockwise direction. U consists in reversing the intermediate counters. In the above solution for 12, it will be seen that 12, 11, and 1, 2, 3 are engaged in the L movement, and 4, 5, 6, 7, 8, 9, 10 in the U movement. The L movement needs 16 moves and U 28, making together 44. We might also involve 10 in the L movement, which would result in L 23, U 21, making also together 44 moves. These I call the first and second methods. But any other scheme will entail an increase of moves. You always get these two methods (of equal economy) for odd or even counters, but the point is to determine just how many to involve in L and how many in U. Here is the solution in table form. But first note, in giving values to n, that 2, 3, and 4 counters are special cases, requiring respectively 3, 3, and 6 moves, and that 5 and 6 counters do not give a minimum solution by the second method—only by the first.

Total No. of Counters	L MOVEMENT		U MOVEMENT		Total No. of Moves
	No. of Counters	No. of Moves	No. of Counters	No. of Moves	
$4n$	$n-1$ and n	$2(n-1)^2+5n-7$	$2n+1$	$2n^2+3n+1$	$4(n^2+n-1)$
$4n-2$	$n-1$ and n	$2(n-1)^2+5n-7$	$2n-1$	$2(n-1)^2+3n-2$	$4n^2-5$
$4n+1$	n and $n+1$	$2n^2+5n-2$	$2n$	$2n^2+3n-4$	$2(2n^2+4n-3)$
$4n-1$	$n-1$ and n	$2(n-1)^2+5n-7$	$2n$	$2n^2+3n-4$	$4n^2+4n-9$

Total No. of Counters	L MOVEMENT		U MOVEMENT		Total No. of Moves
	No. of Counters	No. of Moves	No. of Counters	No. of Moves	
$4n$	n and n	$2n^2+3n-4$	$2n$	$2(n-1)^2+5n-2$	$4(n^2+n-1)$
$4n-2$	$n-1$ and $n-1$	$2(n-1)^2+3n-7$	$2n$	$2(n-1)^2+5n-2$	$4n^2-5$
$4n+1$	n and n	$2n^2+3n-4$	$2n+1$	$2n^2+5n-2$	$2(2n^2+4n-3)$
$4n-1$	n and n	$2n^2+3n-4$	$2n-1$	$2(n-1)^2+5n-7$	$4n^2+4n-9$

More generally we may say that with m counters, where m is even and greater than 4, we require $\frac{m^2 + 4m - 16}{4}$ moves; and where m is odd and greater than 3, $\frac{m^2 + 6m - 31}{4}$ moves. I have thus shown the reader how to find the minimum number of moves for any case, and the character and direction of the moves. I will leave him to discover for himself how the actual order of moves is to be determined. This is a hard nut, and requires careful adjustment of the L and the U movements, so that they may be mutually accommodating.

53. THE TEN APPLES.

Number the plates (1, 2, 3, 4), (5, 6, 7, 8), (9, 10, 11, 12), (13, 14, 15, 16) in successive rows from the top to the bottom. Then transfer the apple from 8 to 10 and play as follows, always removing the apple jumped over: 9–11, 1–9, 13–5, 16–8, 4–12, 12–10, 3–1, 1–9, 9–11.

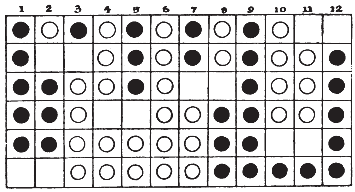
54. THE TWELVE PENNIES.

Here is one of several solutions. Move 12 to 3, 7 to 4, 10 to 6, 8 to 1, 9 to 5, 11 to 2.

55. THE ECCENTRIC CHEESEMONGER.

To leave the three piles at the extreme ends of the rows, the cheeses may be moved as follows—the numbers refer to the cheeses and not to their positions in the row: 7–2, 8–7, 9–8, 10–15, 6–10, 5–6, 14–16, 13–14, 12–13, 3–1, 4–3, 11–4. This is probably the easiest solution of all to find. To get three of the piles on cheeses 13, 14, and 15, play thus: 9–4, 10–9, 11–10, 6–14, 5–6, 12–15, 8–12, 7–8, 16–5, 3–13, 2–3, 1–2. To leave the piles on cheeses 3, 5, 12, and 14, play thus: 8–3, 9–14, 16–12, 1–5, 10–9, 7–10, 11–8, 2–1, 4–16, 13–2, 6–11, 15–4.

56. THE HAT PUZZLE.



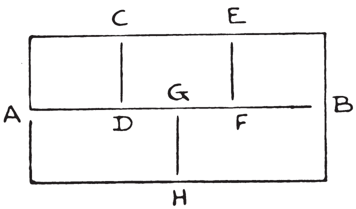
I suggested that the reader should try this puzzle with counters, so I give my solution in that form. The silk hats are represented by black counters and the felt hats by white counters. The first row shows the hats in their original positions, and then each successive row shows how they appear after one of the five manipulations. It will thus be seen that we first move hats 2 and 3, then 7 and 8, then 4 and 5, then 10 and 11, and, finally, 1 and 2, leaving the four silk hats together, the four felt hats together, and the two vacant pegs at one end of the row. The first three pairs moved are dissimilar hats, the last two pairs being similar. There are other ways of solving the puzzle.

57. BOYS AND GIRLS.

There are a good many different solutions to this puzzle. Any contiguous pair, except 7–8, may be moved first, and after the first move there are variations. The following solution shows the position from the start right through each successive move to the end:—

. . 1 2 3 4 5 6 7 8
4 3 1 2 . . 5 6 7 8
4 3 1 2 7 6 5 . . 8
4 3 1 2 7 . . 5 6 8
4 . . 2 7 1 3 5 6 8
4 8 6 2 7 1 3 5 . .

58. A JUVENILE PUZZLE.



As the conditions are generally understood, this puzzle is incapable of solution. This can be demonstrated quite easily. So we have to look for some catch or quibble in the statement of what we are asked to do. Now if you fold the paper and then push the point of your pencil down between the fold, you can with one stroke make the two lines CD and EF in our diagram. Then start at A, and describe the line ending at B. Finally put in the last line GH, and the thing is done strictly within the conditions, since folding the paper is not actually forbidden. Of course the lines are here left unjoined for the purpose of clearness.

In the rubbing out form of the puzzle, first rub out A to B with a single finger in one stroke. Then rub out the line GH with one finger. Finally, rub out the remaining two vertical lines with two fingers at once! That is the old trick.

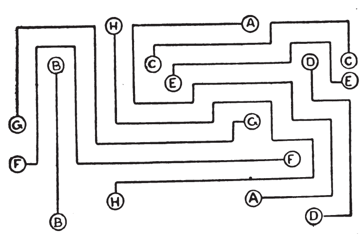
59. THE TUBE INSPECTOR'S PUZZLE.

The inspector need only travel nineteen miles if he starts at B and takes the following route: BADGDEFIFCBEHKLIHGJK. Thus the only portions of line travelled over twice are the two sections D to G and F to I. Of course, the route may be varied, but it cannot be shortened.

60. THE CYCLIST'S TOUR.

When Mr. Maggs replied, "No way, I'm sure," he was not saying that the thing was impossible, but was really giving the actual route by which the problem can be solved. Starting from the star, if you visit the towns in the order, NO WAY, I'M SURE, you will visit every town once, and only once, and end at E. So both men were correct. This was the little joke of the puzzle, which is not by any means difficult.

61. A PUZZLE FOR MOTORISTS.

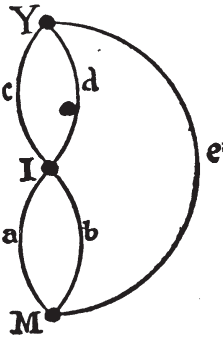


The routes taken by the eight drivers are shown in the illustration, where the dotted line roads are omitted to make the paths clearer to the eye.

62. THE DIAMOND PUZZLE.

There are 252 different ways. The general formula is that, for words of n letters (not palindromes, as in the case of the next puzzle), when grouped in this manner, there are always $2^{(n+1)} - 4$ different readings. This does not allow diagonal readings, such as you would get if you used instead such a word as DIGGING, where it would be possible to pass from one G to another G by a diagonal step.

63. THE MONK AND THE BRIDGES.



The problem of the Bridges may be reduced to the simple diagram shown in illustration. The point M represents the Monk, the point I the Island, and the point Y the Monastery. Now the only direct ways from M to I are by the bridges *a* and *b*; the only direct ways from I to Y are by the bridges *c* and *d*; and there is a direct way from M to Y by the bridge *e*. Now, what we have to do is to count all the routes that will lead from M to Y, passing over all the bridges, *a*, *b*, *c*, *d*, and *e* once and once only. With the simple diagram under the eye it is quite easy, without any elaborate rule, to count these routes methodically. Thus, starting from *a*, *b*, we find there are only two ways of completing the route; with *a, c*, there are only two routes; with *a*, *d*, only two routes; and so on. It will be found that there are sixteen such routes in all, as in the following list:—

a b e c d	*b c d a e*
a b e d c	*b c e a d*
a c d b e	*b d c a e*
a c e b d	*b d e a c*
a d e b c	*e c a b d*

a d c b e	*e c b a d*
b a e c d	*e d a b c*
b a e d c	*e d b a c*

If the reader will transfer the letters indicating the bridges from the diagram to the corresponding bridges in the original illustration, everything will be quite obvious.

64. THOSE FIFTEEN SHEEP.

If we read the exact words of the writer in the cyclopædia, we find that we are not told that the pens were all necessarily empty! In fact, if the reader will refer back to the illustration, he will see that one sheep is already in one of the pens. It was just at this point that the wily farmer said to me, "*Now* I'm going to start placing the fifteen sheep." He thereupon proceeded to drive three from his flock into the already occupied pen, and then placed four sheep in each of the other three pens. "There," says he, "you have seen me place fifteen sheep in four pens so that there shall be the same number of sheep in every pen." I was, of course, forced to admit that he was perfectly correct, according to the exact wording of the question.

65. A PUZZLE FOR CARD-PLAYERS.

In the following solution each of the eleven lines represents a sitting, each column a table, and each pair of letters a pair of partners.

A B—I L	E J—G K	F H—C D
A C—J B	F K—H L	G I—D E
A D—K C	G L—I B	H J—E F
A E—L D	H B—J C	I K—F G
A F—B E	I C—K D	J L—G H
A G—C F	J D—L E	K B—H I

A H—D G	K E—B F	L C—I J
A I—E H	L F—C G	B D—J K
A J—F I	B G—D H	C E—K L
A K—G J	C H—E I	D F—L B
A L—H K	D I—F J	E G—B C

It will be seen that the letters B, C, D ...L descend cyclically. The solution given above is absolutely perfect in all respects. It will be found that every player has every other player once as his partner and twice as his opponent.

66. THE GLASS BALLS.

There are, in all, sixteen balls to be broken, or sixteen places in the order of breaking. Call the four strings A, B, C, and D—order is here of no importance. The breaking of the balls on A may occupy any 4 out of these 16 places—that is, the combinations of 16 things, taken 4 together, will be $\frac{13 \times 14 \times 15 \times 16}{1 \times 2 \times 3 \times 4} = 1{,}820$

ways for A. In every one of these cases B may occupy any 4 out of the remaining 12 places, making $\frac{9 \times 10 \times 11 \times 12}{1 \times 2 \times 3 \times 4} = 495$

ways. Thus 1,820 × 495 = 900,900 different placings are open to A and B. But for every one of these cases C may occupy $\frac{5 \times 6 \times 7 \times 8}{1 \times 2 \times 3 \times 4} = 70$

different places; so that 900,900 × 70 = 63,063,000 different placings are open to A, B, and C. In every one of these cases, D has no choice but to take the four places that remain. Therefore the correct answer is that the balls may be broken in 63,063,000 different ways under the conditions.

67. FIFTEEN LETTER PUZZLE.

The following will be found to comply with the conditions of grouping:—

ALE	MET	MOP	BLM
BAG	CAP	YOU	CLT
IRE	OIL	LUG	LNR
NAY	BIT	BUN	BPR
AIM	BEY	RUM	GMY
OAR	GIN	PLY	CGR
PEG	ICY	TRY	CMN
CUE	COB	TAU	PNT
ONE	GOT	PIU	

The fifteen letters used are A, E, I, O, U, Y, and B, C, G, L, M, N, P, R, T. The number of words is 27, and these are all shown in the first three columns. The last word, PIU, is a musical term in common use; but although it has crept into some of our dictionaries, it is Italian, meaning "a little; slightly." The remaining twenty-six are good words. Of course a TAU-cross is a T-shaped cross, also called the cross of St. Anthony, and borne on a badge in the Bishop's Palace at Exeter. It is also a name for the toad-fish.

We thus have twenty-six good words and one doubtful, obtained under the required conditions, and I do not think it will be easy to improve on this answer. Of course we are not bound by dictionaries but by common usage. If we went by the dictionary only in a case of this kind, we should find ourselves involved in prefixes, contractions, and such absurdities as I.O.U., which Nuttall actually gives as a word.

68. THE MOUSE-TRAP PUZZLE.

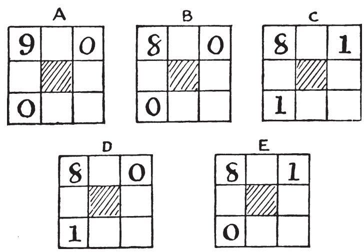
If we interchange cards 6 and 13 and begin our count at 14, we may take up all the twenty-one cards—that is, make twenty-one "catches"—in the following order: 6, 8, 13, 2, 10, 1, 11, 4, 14, 3, 5, 7, 21, 12, 15, 20, 9, 16, 18, 17, 19. We may also exchange 10 and 14 and start at 16, or exchange 6 and 8 and start at 19.

69. THE EIGHT VILLAS.

There are several ways of solving the puzzle, but there is very little difference between them. The solver should, however, first of all bear in mind that in making his calculations he need only consider the four villas that stand at the corners, because the intermediate villas can never vary when the corners are known. One way is to place the numbers nought to 9 one at a time in the top left-hand corner, and then consider each case in turn.

Now, if we place 9 in the corner as shown in the Diagram A, two of the corners cannot be occupied, while the corner that is diagonally opposite may be filled by 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9 persons. We thus see that there are 10 solutions with a 9 in the corner. If, however, we substitute 8, the two corners in the same row and column may contain 0, 0, or 1, 1, or 0, 1, or 1, 0. In the case of B, ten different selections may be made for the fourth corner; but in each of the cases C, D, and E, only nine selections are possible, because we cannot use the 9. Therefore with 8 in the top left-hand corner there are 10 + (3 × 9) = 37 different solutions. If we then try 7 in the corner, the result will be 10 + 27 + 40, or 77 solutions. With 6 we get 10 + 27 + 40 + 49 = 126; with 5, 10 + 27 + 40 + 49 + 54 = 180; with 4, the same as with 5, + 55 = 235 ; with 3, the same as with 4, + 52 = 287; with 2, the same as with 3, + 45 = 332; with 1, the same as with 2, + 34 = 366, and with nought in the top left-hand corner the number of solutions will be found to be 10 + 27 + 40 + 49 + 54 + 55 + 52 + 45 + 34 + 19 = 385. As there is no other number to be placed in the top left-hand corner, we have now only to add these totals together thus, 10 + 37 + 77 + 126 + 180 + 235 + 287 + 332 + 366 + 385 = 2,035. We therefore find that the total number of ways in which tenants may occupy some or all of the eight villas so that there shall be always nine persons living along each side of the square is 2,035. Of course, this method must obviously cover all the reversals and reflections, since each corner in turn is occupied by every number in all possible combinations with the other two corners that are in line with it.

Here is a general formula for solving the puzzle: $\frac{(n^2 + 3n + 2)(n^2 + 3n + 3)}{6}$. Whatever may be the stipulated number of residents along each of the sides (which number is represented by *n*), the total number of different arrangements may be thus ascertained. In our particular case the number of residents was nine.

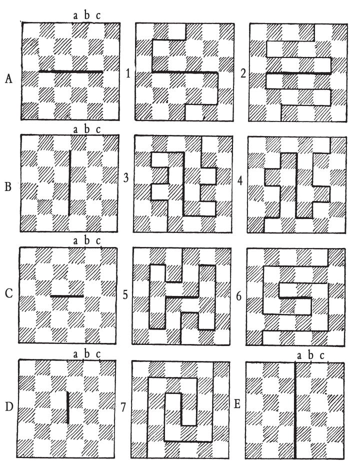


Therefore (81 + 27 + 2) × (81 + 27 + 3) and the product, divided by 6, gives 2,035. If the number of residents had been 0, 1, 2, 3, 4, 5, 6, 7, or 8, the total arrangements would be 1, 7, 26, 70, 155, 301, 532, 876, or 1,365 respectively.

70. CHEQUERED BOARD DIVISIONS.

There are 255 different ways of cutting the board into two pieces of exactly the same size and shape. Every way must involve one of the five cuts shown in

Diagrams A, B, C, D, and E. To avoid repetitions by reversal and reflection, we need only consider cuts that enter at the points *a*, *b*, and *c*. But the exit must always be at a point in a straight line from the entry through the centre. This is the most important condition to remember. In case B you cannot enter at *a*, or you will get the cut provided for in E. Similarly in C or D, you must not enter the key-line in the same direction as itself, or you will get A or B. If you are working on A or C and entering at a, you must consider joins at one end only of the key-line, or you will get repetitions. In other cases you must consider joins at both ends of the key; but after leaving a in case D, turn always either to right or left—use one direction only. Figs. 1 and 2 are examples under A; 3 and 4 are examples under B; 5 and 6 come under C;



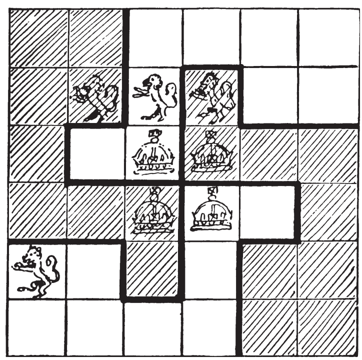
and 7 is a pretty example of D. Of course, E is a peculiar type, and obviously admits of only one way of cutting, for you clearly cannot enter at *b* or *c*.

Here is a table of the results:—

	a	*b*	*c*	Ways.
A =	8 +	17 +	21 =	46
B =	0 +	17 +	21 =	38
C =	15 +	31 +	39 =	85
D =	17 +	29 +	39 =	85
E =	1 +	0 +	0 =	1
	41	94	120	255

I have not attempted the task of enumerating the ways of dividing a board 8 × 8—that is, an ordinary chessboard. Whatever the method adopted, the solution would entail considerable labour.

71. LIONS AND CROWNS.



Here is the solution. It will be seen that each of the four pieces (after making the cuts along the thick lines) is of exactly the same size and shape, and that each piece contains a lion and a crown. Two of the pieces are shaded so as to make the solution quite clear to the eye.

72. BOARDS WITH AN ODD NUMBER OF SQUARES.

There are fifteen different ways of cutting the 5 × 5 board (with the central square removed) into two pieces of the same size and shape. Limitations of space will not allow me to give diagrams of all these, but I will enable the reader to draw them all out for himself without the slightest difficulty. At whatever point on the edge your cut enters, it must always end at a point on the edge, exactly opposite in a line through the centre of the square. Thus, if you enter at point 1 (see Fig. 1) at the top, you must leave at point 1 at the bottom. Now, 1 and 2 are the only two really different points of entry; if we use any others they will simply produce similar solutions. The directions of the cuts in the following fifteen

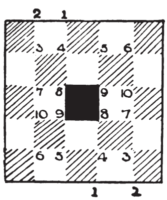


Fig. 1

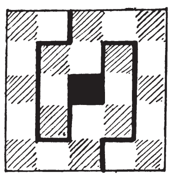


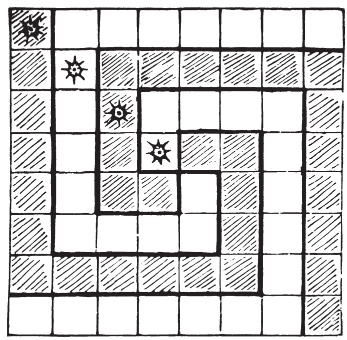
Fig.2

solutions are indicated by the numbers on the diagram. The duplication of the numbers can lead to no confusion, since every successive number is contiguous to the previous one. But whichever direction you take from the top downwards you must repeat from the bottom upwards, one direction being an exact reflection of the other.

1, 4, 8.
1, 4, 3, 7, 8.
1, 4, 3, 7, 10, 9.
1, 4, 3, 7, 10, 6, 5, 9.
1, 4, 5, 9.
1, 4, 5, 6, 10, 9.
1, 4, 5, 6, 10, 7, 8.
2, 3, 4, 8.
2, 3, 4, 5, 9.
2, 3, 4, 5, 6, 10, 9.
2, 3, 4, 5, 6, 10, 7, 8.
2, 3, 7, 8.
2, 3, 7, 10, 9.
2, 3, 7, 10, 6, 5, 9.
2, 3, 7, 10, 6, 5, 4, 8.

It will be seen that the fourth direction (1, 4, 3, 7, 10, 6, 5, 9) produces the solution shown in Fig. 2. The thirteenth produces the solution given in propounding the puzzle, where the cut entered at the side instead of at the top. The pieces, however, will be of the same shape if turned over, which, as it was stated in the conditions, would not constitute a different solution.

73. THE GRAND LAMA'S PROBLEM.



The method of dividing the chessboard so that each of the four parts shall be of exactly the same size and shape, and contain one of the gems, is shown in the diagram. The method of shading the squares is adopted to make the shape of the pieces clear to the eye. Two of the pieces are shaded and two left white.

74. THE EIGHT ROOKS.

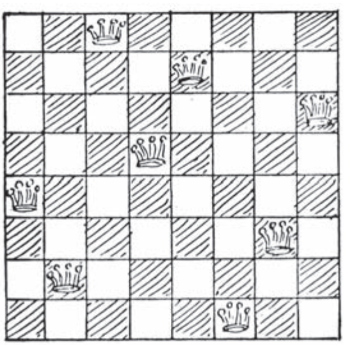
Obviously there must be a rook in every row and every column. Starting with the top row, it is clear that we may put our first rook on any one of eight different squares. Wherever it is placed, we have the option of seven squares for the second rook in the second row. Then we have six squares from which to select the third row, five in the fourth, and so on. Therefore the number of our different ways must be 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 40,320 (that is |8), which is the correct answer.

How many ways there are if mere reversals and reflections are not counted as different has not yet been determined; it is a difficult problem. But this point, on a smaller square, is considered in the next puzzle.

75. THE FOUR LIONS.

There are only seven different ways under the conditions. They are as follows: 1 2 3 4, 1 2 4 3, 1 3 2 4, 1 3 4 2, 1 4 3 2, 2 1 4 3, 2 4 1 3. Taking the last example, this notation means that we place a lion in the second square of first row, fourth square of second row, first square of third row, and third square of fourth row. The first example is, of course, the one we gave when setting the puzzle.

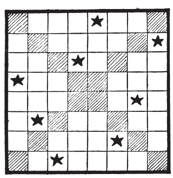
76. THE EIGHT QUEENS.

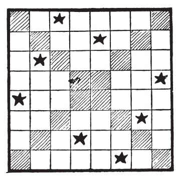


The solution to this puzzle is shown in the diagram. It will be found that no queen attacks another, and also that no three queens are in a straight line in any oblique direction. This is the only arrangement out of the twelve fundamentally different ways of placing eight queens without attack that fulfils the last condition.

77. THE EIGHT STARS.

The solution of this puzzle is shown in the first diagram. It is the only possible solution within the conditions stated. But if one of the eight stars had not already been placed as shown, there would then have been eight ways of arranging the stars according to this scheme, if we count reversals and reflections as different. If you turn this page round so that each side is in turn at the bottom, you will get the four reversals; and if you reflect each of these in a mirror, you will get the four reflections. These are, therefore, merely eight aspects of one "fundamental solution." But without that first star being so placed, there is another fundamental solution, as shown in the second diagram. But this arrangement being in a way symmetrical, only produces four different aspects by reversal and reflection.





78. QUEENS AND BISHOP PUZZLE.

The bishop is on the square originally occupied by the rook, and the four queens are so placed that every square is either occupied or attacked by a piece. (Fig. 1.)

I pointed out in 1899 that if four queens are placed as shown in the diagram (Fig. 2), then the fifth queen may be placed on

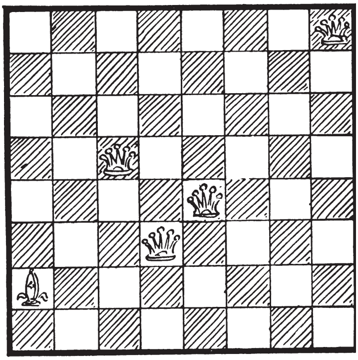


Fig. 1

any one of the twelve squares marked a, b, c, d, and e; or a rook on the two squares, c; or a bishop on the eight squares, a, b, and e; or a pawn on the square b; or a king on the four squares, b, c, and e. The only known arrangement for four queens and a knight is that given by Mr. J. Wallis in *The Strand Magazine* for August 1908, here reproduced. (Fig. 3.)

I have recorded a large number of solutions with four queens and a rook, or bishop, but the only arrangement, I believe, with three queens and two rooks in which all the pieces are guarded is that of which I give an illustration (Fig. 4), first published by Dr. C. Planck. But I

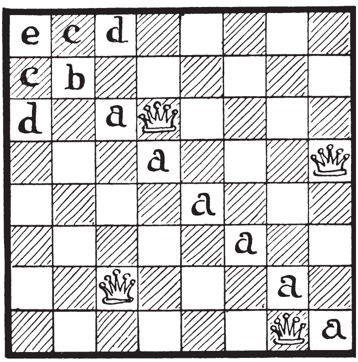


Fig. 2

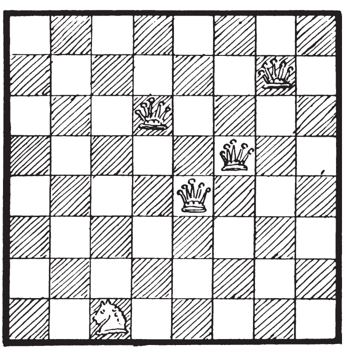


Fig. 3

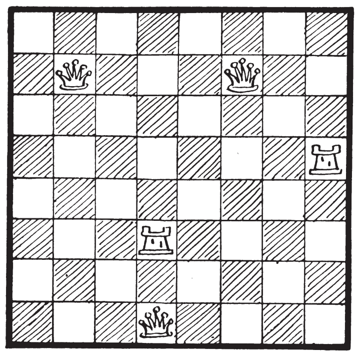


Fig. 4

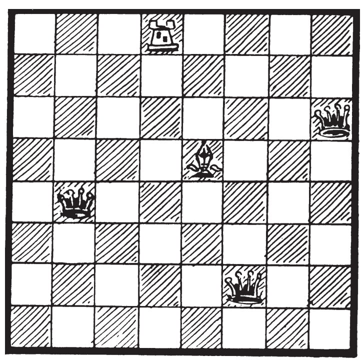
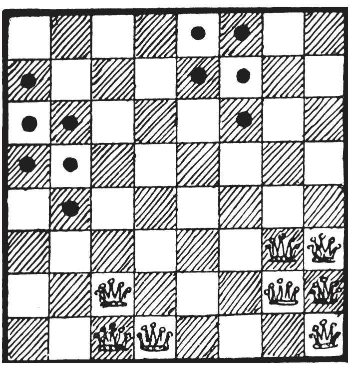


Fig. 5

have since found the accompanying solution with three queens, a rook, and a bishop, though the pieces do not protect one another. (Fig. 5.)

79. THE AMAZONS.

It will be seen that only three queens have been removed from their positions on the edge of the board, and that, as a consequence, eleven squares (indicated by the black dots) are left unattacked by any queen. I will hazard the statement that eight queens cannot be placed on the chessboard so as to leave more than eleven squares unattacked. It is true that we have no rigid proof of this yet, but I have entirely convinced myself of the truth of the statement. There are at least five different ways of arranging the queens so as to leave eleven squares unattacked.



80. THE KNIGHT-GUARDS

The smallest possible number of knights with which this puzzle can be solved is fourteen. It has sometimes been assumed that there are a great many different solutions. As a matter of fact, there are only three arrangements—not counting mere reversals and reflections as different. Curiously enough, nobody seems ever to have hit on the following simple proof, or to have thought of dealing with the black and the white squares separately.

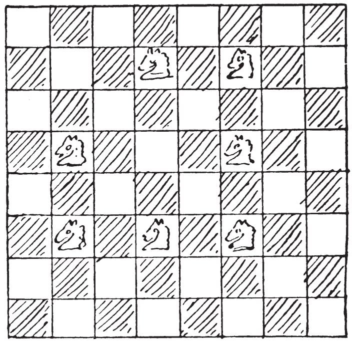


Diagram 1.

Seven knights can be placed on the board on white squares so as to attack every black square in two ways only. These are shown in Diagrams 1 and 2. Note that three knights occupy the same position in both arrangements. It is therefore clear that if we turn the board so that a black square shall

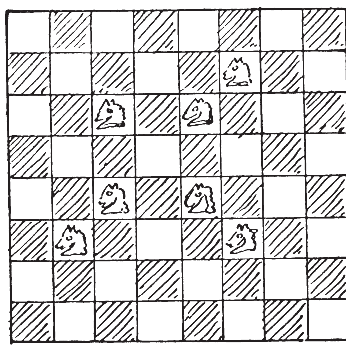


Diagram 2.

be in the top left-hand corner instead of a white, and place the knights in exactly the same positions, we shall have two similar ways of attacking all the white squares. I will assume the reader has made the two last described diagrams on transparent paper, and marked them 1*a* and 2*a*. Now,

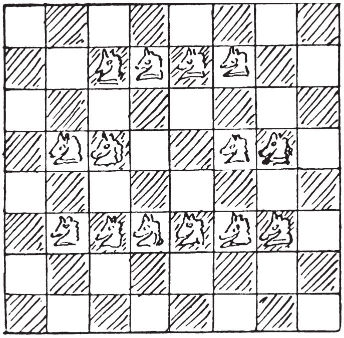


Diagram 3.

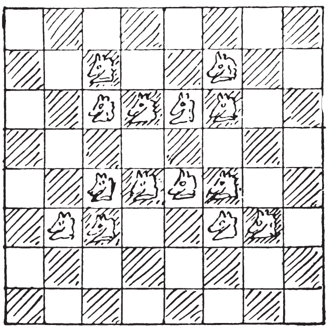


Diagram 4.

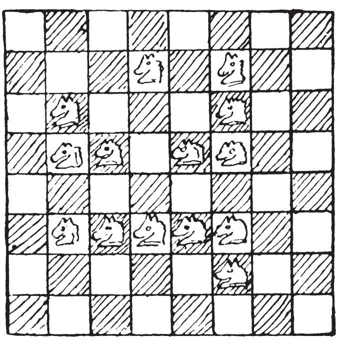
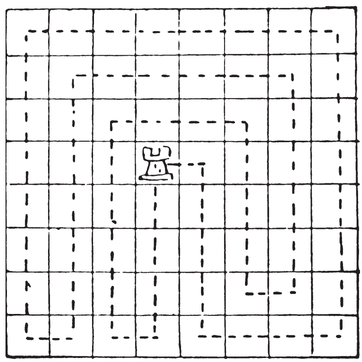


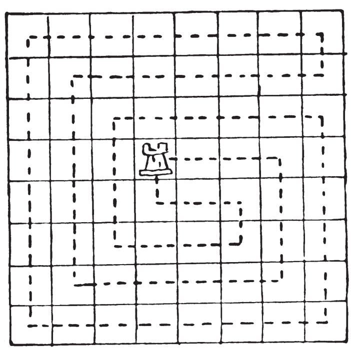
Diagram 5.

by placing the transparent Diagram 1*a* over 1 you will be able to obtain the solution in Diagram 3, by placing 2*a* over 2 you will get Diagram 4, and by placing 2*a* over 1 you will get Diagram 5. You may now try all possible combinations of those two pairs of diagrams, but you will only get the three arrangements I have given, or their reversals and reflections. Therefore these three solutions are all that exist.

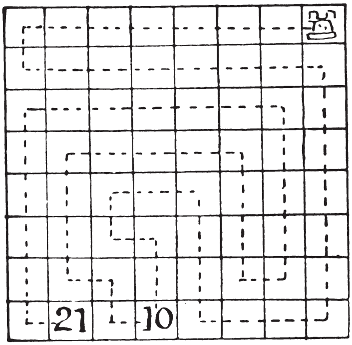
81. THE ROOK'S TOUR.

The only possible minimum solutions are shown in the two diagrams, where it will be seen that only sixteen moves are required to perform the feat. Most people find it difficult to reduce the number of moves below seventeen.



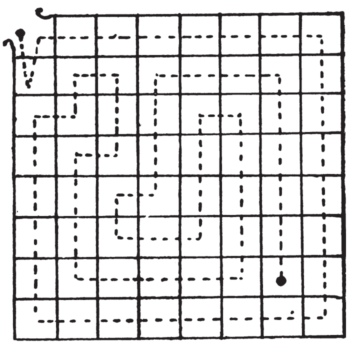


82. THE ROOK'S JOURNEY.



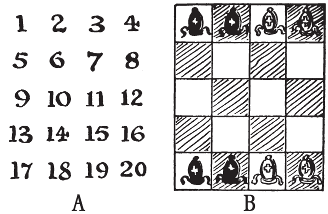
I show the route in the diagram. It will be seen that the tenth move lands us at the square marked "10," and that the last move, the twenty-first, brings us to a halt on square "21."

83. THE LANGUISHING MAIDEN.



The dotted line shows the route in twenty-two straight paths by which the knight may rescue the maiden. It is necessary, after entering the first cell, immediately to return before entering another. Otherwise a solution would not be possible.

84. A NEW BISHOP'S PUZZLE.

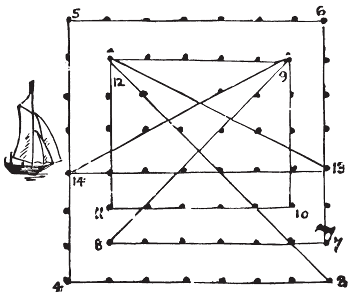


Play as follows, using the notation indicated by the numbered squares in Diagram A:—

	White	Black
1.	18 – 15	3 – 6
2.	17 – 8	4 – 13
3.	19 – 14	2 – 7
4.	15 – 5	6 – 16
5.	8 – 3	13 – 18
6.	14 – 9	7 – 12
7.	5 – 10	16 – 11
8.	9 – 19	12 – 2
9.	10 – 4	11 – 17
10.	20 – 10	1 – 11
11.	3 – 9	18 – 12
12.	10 – 13	11 – 8
13.	19 – 16	2 – 5
14.	16 – 1	5 – 20
15.	9 – 6	12 – 15
16.	13 – 7	8 – 14
17.	6 – 3	15 – 18
18.	7 – 2	14 – 19

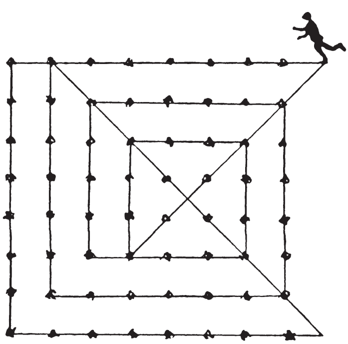
Diagram B shows the position after the ninth move. Bishops at 1 and 20 have not yet moved, but 2 and 19 have sallied forth and returned. In the end, 1 and 19, 2 and 20, 3 and 17, and 4 and 18 will have exchanged places. Note the position after the thirteenth move.

85. THE YACHT RACE.



The diagram explains itself. The numbers will show the direction of the lines in their proper order, and it will be seen that the seventh course ends at the flag-buoy, as stipulated.

86. THE SCIENTIFIC SKATER.



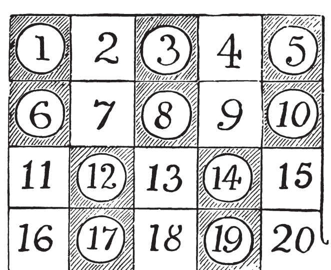
In this case we go beyond the boundary of the square. Apart from that, the moves are all queen moves. There are three or four ways in which it can be done.

Here is one way of performing the feat:—

It will be seen that the skater strikes out all the stars in one continuous journey of fourteen straight lines, returning to the point from which he started. To follow the skater's course in the diagram it is necessary always to go as far as we can in a straight line before turning.

87. THE GREYHOUND PUZZLE.

There are several interesting points involved in this question. In the first place, if we had made no stipulation as to the positions of the two ends of the string, it is quite impossible to form any such string unless we begin and end in the top and bottom row of kennels. We may begin in the top row and end in the bottom (or, of course, the reverse), or we may begin in one of these rows and end in the same. But we can never begin or end in one of the two central rows. Our places of starting and ending, however, were fixed for us. Yet the first half of our route must be confined entirely to those squares that are distinguished in the following diagram by circles, and the second half will therefore be confined to the squares that are not circled. The squares reserved for the two half-strings will be seen to be symmetrical and similar.



The next point is that the first half-string must end in one of the central rows, and the second half-string must begin in one of these rows. This is now obvious, because they have to link together to form the complete string, and every square on an outside row is connected by a knight's move with similar squares only—that is, circled or non-circled as the case may be. The half-strings can, therefore, only be linked in the two central rows.

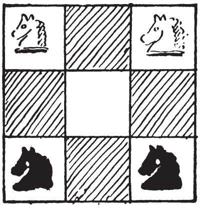
Now, there are just eight different first half-strings, and consequently also eight second half-strings. We shall see that these combine to form twelve complete strings, which is the total number that exist and the correct solution of our puzzle. I do not propose to give all the routes at length, but I will so far indicate them that if the reader has dropped any he will be able to discover which they are and work them out for himself without any difficulty. The following numbers apply to those in the above diagram.

The eight first half-strings are: 1 to 6 (2 routes); 1 to 8 (1 route); 1 to 10 (3 routes); 1 to 12 (1 route); and 1 to 14 (1 route). The eight second half-strings are: 7 to 20 (1 route); 9 to 20 (1 route); 11 to 20 (3 routes); 13 to 20 (1 route); and 15 to 20 (2 routes). Every different way in which you can link one half-string to another gives a different solution. These linkings will be found to be as follows: 6 to 13 (2 cases); 10 to 13 (3 cases); 8 to 11 (3 cases); 8 to 15 (2 cases); 12 to 9 (1 case); and 14 to 7 (1 case). There are, therefore, twelve different linkings and twelve different answers to the puzzle. The route given in the illustration with the greyhound will be found to consist of one of the three half-strings 1 to 10, linked to the half-string 13 to 20. It should be noted that ten of the solutions are produced by five distinctive routes and their reversals—that is, if you indicate these five routes by lines and then turn the diagrams upside down you will get the five other routes. The remaining two solutions are symmetrical (these are the cases where 12 to 9 and 14 to 7 are the links), and consequently they do not produce new solutions by reversal.

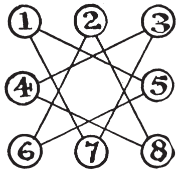
88. THE FOUR FROGS.

The fewest possible moves, counting every move separately, are sixteen. But the puzzle may be solved in seven plays, as follows, if any number of successive moves by one frog count as a single play. All the moves contained within a bracket are a single play; the numbers refer to the toadstools: (1–5), (3–7, 7–1), (8–4, 4–3, 3–7), (6–2, 2–8, 8–4, 4–3), (5–6, 6–2, 2–8), (1–5, 5–6), (7–1).

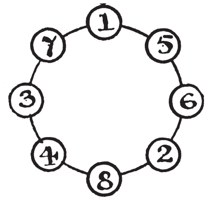
This is the familiar old puzzle by Guarini, propounded in 1512, and I give it here in order to explain my "buttons and string" method of solving this class of moving-counter problem. Diagram A shows the old way of presenting Guarini's puzzle, the point being to make the white knights change places with the black ones. In "The Four Frogs" presentation of the idea the possible directions of the moves are indicated by lines, to obviate the necessity of the reader's understanding the nature of the knight's move in chess. But it will at once be seen that the two problems are identical. The central square can, of course, be ignored, since no knight can ever enter it. Now, regard the toadstools as buttons and the connecting lines as strings, as in Diagram B. Then by disentangling these strings we can clearly present the diagram in the form shown in Diagram C, where the relationship between the buttons is precisely the same as in B. Any solution on C will be applicable to B, and to A. Place your



A

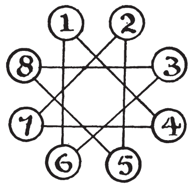


B

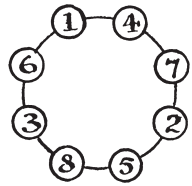


C

D



E



white knights on 1 and 3 and your black knights on 6 and 8 in the C diagram, and the simplicity of the solution will be very evident. You have simply to move the knights round the circle in one direction or the other. Play over the moves given above, and you will find that every little difficulty has disappeared.

In Diagram D I give another familiar puzzle that first appeared in a book published in Brussels in 1789, *Les Petites Aventures de Jerome Sharp*. Place seven counters on seven of the eight points in the following manner. You must always touch a point that is vacant with a counter, and then move it along a straight line leading from that point to the next vacant point (in either direction), where you deposit the counter. You proceed in the same way until all the counters are placed. Remember you always touch a vacant place and slide the counter from it to the next place, which must be also vacant. Now, by the "buttons and string" method of simplification we can transform the diagram into E. Then the solution becomes obvious. "Always move *to* the point that you last moved *from*." This is not, of course, the only way of placing the counters, but it is the simplest solution to carry in the mind.

There are several puzzles in this book that the reader will find lend themselves readily to this method.

89. COUNTING THE RECTANGLES.

There are 1,296 different rectangles in all, 204 of which are squares, counting the square board itself as one, and 1,092 rectangles that are not squares. The general formula is that a board of n^2 squares contains $\frac{(n^2 + n)^2}{4}$ rectangles, of which $\frac{2n^3 + 3n^2 + n}{6}$ are squares and (3n^4 + $\frac{3n^4 + 2n^3 - 3n^2 - 2n}{12}$ are rectangles that are not squares. It is curious and interesting that the total number of rectangles is always the square of the triangular number whose side is *n*.

90. THE ROOKERY.

The answer involves the little point that in the final position the numbered rooks must be in numerical order in the direction contrary to that in which they appear in the original diagram, otherwise it cannot be solved. Play the rooks in the following order of their numbers. As there is never more than one square to which a rook can move (except on the final move), the notation is obvious–5, 6, 7, 5, 6, 4, 3, 6, 4, 7, 5, 4, 7, 3, 6, 7, 3, 5, 4, 3, 1, 8, 3, 4, 5, 6, 7, 1, 8, 2, 1, and rook takes bishop, checkmate. These are the fewest possible moves—thirty-two. The Black king's moves are all forced, and need not be given.

91. THE FORSAKEN KING.

Play as follows:—

White.	Black.
1. P to K 4th	1. Any move
2. Q to Kt 4th	2. Any move except on KB file (*a*)
3. Q to Kt 7th	3. K moves to royal row
4. B to Kt 5th	4. Any move
5. Mate in two moves	
If 3. K other than to royal row	
4. P to Q 4t	4. Any move
5. Mate in two moves	
(*a*) If 2. Any move on KB file	
3. Q to Q 7th	3. K moves to royal row
4. P to Q Kt 3rd	4. Any move
5. Mate in two moves	
If 3. K other than to royal row	
4. P to Q 4th	4. Any move
5. Mate in two moves	

Of course, by "royal row" is meant the row on which the king originally stands at the beginning of a game. Though, if Black plays badly, he may, in certain positions, be mated in fewer moves, the above provides for every variation he can possibly bring about.

92. CHECKMATE!

Remove the White pawn from B 6th to K 4th and place a Black pawn on Black's KB 2nd. Now, White plays P to K 5th, check, and Black must play P to B 4th. Then White plays P takes P *en passant*, checkmate. This was therefore White's last move, and leaves the position given. It is the only possible solution.

93. ANCIENT CHINESE PUZZLE.

Play as follows:—

1. R—Q 6
2. K—R 7
3. R (R 6)—B 6 (mate).

Black's moves are forced, so need not be given.

94. THE MONSTROSITY.

White	Black,
1. P to KB 4	P to QB 3
2. K to B 2	Q to R 4
3. K to K 3	K to Q sq
4. P to B 5	K to B 2
5. Q to K sq	K to Kt 3
6. Q to Kt 3	Kt to QR 3
7. Q to Kt 8	P to KR 4
8. Kt to KB 3	R to R 3
9. Kt to K 5	R to Kt 3
10. Q takes B	R to Kt 6, ch
11. P takes R	K to Kt 4
12. R to R 4	P to B 3
13. R to Q 4	P takes Kt
14. P to QKt 4	P takes R, ch
15. K to B 4	P to R 5
16. Q to K 8	P to R 6
17. Kt to B 3, ch	P takes Kt
18. B to R 3	P to R 7
19. R to Kt sq	P to R 8 (Q)
20. R to Kt 2	P takes R
21. K to Kt 5	Q to KKt 8
22. Q to R 5	K to R 5
23. P to Kt 5	R to B sq
24. P to Kt 6	R to B 2
25. P takes R	P to Kt 8 (B)

26. P to B 8 (R)	Q to B 2
27. B to Q 6	Kt to Kt 5
28. K to Kt 6	K to R 6
29. R to R 8	K to Kt 7
30. P to R 4	Q (Kt 8) to Kt 3
31. P to R 5	K to B 8
32. P takes Q	K to Q 8
33. P takes Q	K to K 8
34. K to B 7	Kt to KR 3, ch
35. K to K 8	B to R 7
36. P to B 6	B to Kt sq
37. P to B 7	K takes B
38. P to B 8 (B)	Kt to Q 4
39. B to Kt 8	Kt to B 3, ch
40. K to Q 8	Kt to K sq
41. P takes Kt (R)	Kt to B 2, ch
42. K to B 7	Kt to Q sq
43. Q to B 7, ch	K to Kt 8

And the position is reached.

The order of the moves is immaterial, and this order may be greatly varied. But, although many attempts have been made, nobody has succeeded in reducing the number of my moves.

95. CROSSING THE STREAM.

First, the two sons cross, and one returns Then the man crosses and the other son returns. Then both sons cross and one returns. Then the lady crosses and the other son returns Then the two sons cross and one of them returns for the dog. Eleven crossings in all.

It would appear that no general rule can be given for solving these river-crossing puzzles. A formula can be found for a particular case (say on No. 375 or 376) that would apply to any number of individuals under the restricted conditions; but it is not of much use, for some little added stipulation will entirely upset it. As in the case of the measuring puzzles, we generally have to rely on individual ingenuity.

96. CROSSING THE RIVER AXE.

Here is the solution:—

G, J, and T stand for Giles, Jasper, and Timothy; and 8, 5, 3, for £800, £500, and £300 respectively. The two side columns represent the left bank and the right bank, and the middle column the river. Thirteen crossings are necessary, and each line shows the position when the boat is in mid-stream during a crossing, the point of the bracket indicating the direction.

It will be found that not only is no person left alone on the land or in the boat with more than his share of the spoil, but that also no two persons are left with more than their joint shares, though this last point was not insisted upon in the conditions.

	...{J 5)...	...G T 8 3
5...	...(J }...	...G T 8 3
5...	...{G 3)...	...J T 8
53...	...(G }...	...J T 8
53...	...{J T)...	...G 8
J 5...	...(T 3}...	...G 8
J 5...	...{G 8)...	...T 3
G 8...	...(J 5}...	...T 3
G 8...	...{ J T)...	...53
J T 8...	...(}...	...53
J T 8...	...{G 3)...	...5
G T 8 3...	...(J }...	...5
G T 8 3...	...{J 5)...	

97. FIVE JEALOUS HUSBANDS.

It is obvious that there must be an odd number of crossings, and that if the five husbands had not been jealous of one another the party might have all got over

	ABCDE abcde★	..	
1.	ABCDE de	..	★ abc
2.	ABCDE bcde★	..	a
3.	ABCDE e	..	★ abcde
4.	ABCDE de★	..	abc
5.	DE de	..	★ ABC abc
6.	CDE cde★	..	AB ab
7.	cde	..	★ ABCDE ab
8.	bcde★	..	ABCDE a
9.	e	..	★ ABCDE abcde
10.	bc e★	..	ABCDE a d
11.		..	★ ABCDE abcde

in nine crossings. But no wife was to be in the company of a man or men unless her husband was present. This entails two more crossings, eleven in all.

The following shows how it might have been done. The capital letters stand for the husbands, and the small letters for their respective wives. The position of affairs is shown at the start, and after each crossing between the left bank and the right, and the boat is represented by the asterisk. So you can see at a glance that a, b, and c went over at the first crossing, that b and c returned at the second crossing, and so on.

There is a little subtlety concealed in the words "show the *quickest* way."

Everybody correctly assumes that, as we are told nothing of the rowing capabilities of the party, we must take it that they all row equally well. But it is obvious that two such persons should row more quickly than one.

Therefore in the second and third crossings two of the ladies should take back the boat to fetch d, not one of them only. This does not affect the number of landings, so no time is lost on that account. A similar opportunity occurs in crossings 10 and 11, where the party again had the option of sending over two ladies or one only.

To those who think they have solved the puzzle in nine crossings I would say that in every case they will find that they are wrong. No such jealous husband would, in the circumstances, send his wife over to the other bank to a man or men, even if she assured him that she was coming back next time in the boat. If readers will have this fact in mind, they will at once discover their errors.

98. THE FOUR ELOPEMENTS.

If there had been only three couples, the island might have been dispensed with, but with four or more couples it is absolutely necessary in order to cross under the conditions laid down. It can be done in seventeen passages from land to land (though French mathematicians have declared in their books that in such circumstances twenty-four are needed), and it cannot be done in fewer. I will give one way. A, B, C, and D are the young men, and a, b, c, and d are the girls to whom they are respectively engaged. The three columns show the positions of the different individuals on the lawn, the island, and the opposite shore before starting and after each passage, while the asterisk indicates the position of the boat on every occasion.

Having found the fewest possible passages, we should consider two other points in deciding on the "quickest method": Which persons were the most expert in handling the oars, and which method entails the fewest possible delays in getting in and out of the boat? We have no data upon which to decide the first point, though it is probable that, as the boat belonged to the girls' household, they would be capable oarswomen. The other point, however, is important, and in the solution I have given (where the girls do 8-13ths of the rowing and A and D need not row at all) there are only sixteen gettings-in and sixteen gettings-out. A man and a girl are never in the boat together, and no man ever lands on the island. There are other methods that require several more exchanges of places.

Lawn	Island	Shore
ABCD abcd ⋆		
ABCD cd		ab ⋆
ABCD bcd ⋆		a
ABCD d	bc ⋆	a
ABCD cd ⋆	b	a
D cd	b	AB a ⋆
BCD cd ⋆	b	A a
BCD	bcd	A a
BCD d ⋆	bc	A a
D d	bc	ABC a ⋆
D d	abc ⋆	ABC
D d	b ⋆	ABC a c ⋆
B D d ⋆	b ⋆	A C a c
d	b ⋆	ABCD a c ⋆
d	bc ⋆	ABCD a
cd ⋆		ABCD abc ⋆
		ABCD ab
		ABCD abcd ⋆

99. STEALING THE CASTLE TREASURE.

Here is the best answer, in eleven manipulations:—

Treasure down.
Boy down—treasure up.
Youth down—boy up.
Treasure down.
Man down—youth and treasure up.
Treasure down.
Boy down—treasure up.
Treasure down.
Youth down—boy up.
Boy down—treasure up.
Treasure down.

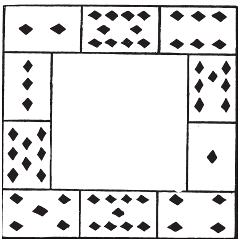
100. DOMINOES IN PROGRESSION.

There are twenty-three different ways. You may start with any domino, except the 4—4 and those that bear a 5 or 6, though only certain initial dominoes may be played either way round. If you are given the common difference and the first domino is played, you have no option as to the other dominoes. Therefore all I need do is to give the initial domino for all the twenty-three ways, and state the common difference. This I will do as follows:—

With a common difference of 1, the first domino may be either of these: 0—0, 0—1, 1—0, 0—2, 1—1, 2—0, 0—3, 1—2, 2—1, 3—0, 0—4, 1—3, 2—2, 3—1, 1—4, 2—3, 3—2, 2—4, 3—3, 3—4. With a difference of 2, the first domino may be 0—0, 0—2, or 0—1. Take the last case of all as an example. Having played the 0—1, and the difference being 2, we are compelled to continue with 1—2, 2—3, 3—4. 4—5, 5—6. There are three dominoes that can never be used at all. These are 0—5, 0—6, and 1—6. If we used a box of dominoes extending to 9—9, there would be forty different ways.

101. THE CARD FRAME PUZZLE.

The sum of all the pips on the ten cards is 55. Suppose we are trying to get 14 pips on every side. Then 4 times 14 is 56. But each of the four corner cards is added in twice, so that 55 deducted from 56, or 1, must represent the sum of the four corner cards. This is clearly impossible; therefore 14 is also impossible. But suppose we came to trying 18. Then 4 times 18 is 72, and if we deduct 55 we get 17 as the sum of the corners. We need then only try different arrangements with the four corners always summing to 17, and we soon discover the following solution:—



The final trials are very limited in number, and must with a little judgment either bring us to a correct solution or satisfy us that a solution is impossible under the conditions we are attempting. The two centre cards on the upright sides can, of course, always be interchanged, but I do not call these different solutions. If you reflect in a mirror you get another arrangement, which also is not considered different. In the answer given, however, we may exchange the 5 with the 8 and the 4 with the 1. This is a different solution. There are two solutions with 18, four with 19, two with 20, and two with 22—ten arrangements in all. Readers may like to find all these for themselves.

102. THE CROSS OF CARDS.

There are eighteen fundamental arrangements, as follows, where I only give the numbers in the horizontal bar, since the remainder must naturally fall into their places.

5 6 1 7 4   2 4 5 6 8
3 5 1 6 8   3 4 5 6 7
3 4 1 7 8   1 4 7 6 8
2 5 1 7 8   2 3 7 6 8
2 5 3 6 8   2 4 7 5 8
1 5 3 7 8   3 4 9 5 6
2 4 3 7 8   2 4 9 5 7
1 4 5 7 8   1 4 9 6 7
2 3 5 7 8   2 3 9 6 7

It will be noticed that there must always be an odd number in the centre, that there are four ways each of adding up 23, 25, and 27, but only three ways each of summing to 24 and 26.

103. 'STRAND' PATIENCE.

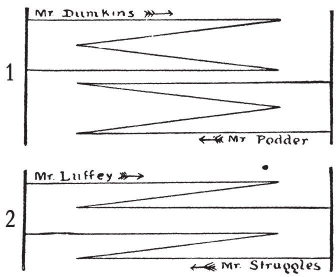
The reader may find a solution quite easy in a little over 200 moves, but, surprising as it may at first appear, not more than 62 moves are required. Here is the play: By "4 C up" I mean a transfer of the 4 of clubs with all the cards that rest on it. 1 D on space, 2 S on space, 3 D on space, 2 S on 3 D, 1 H on 2 S, 2 C on space, 1 D on 2 C, 4 S on space, 3 H on 4 S (9 moves so far), 2 S up on 3 H (3 moves), 5 H and 5 D exchanged, and 4 C on 5 D (6 moves), 3 D on 4 C (1), 6 S (with 5 H) on space (3), 4 C up on 5 H (3), 2 C up on 3 D (3), 7 D on space (1), 6 C up on 7 D (3), 8 S on space (1), 7 H on 8 S (1), 8 C on 9 D (1), 7 H on 8 C (1), 8 S on 9 H (1), 7 H on 8 S (1), 7 D up on 8 C (5), 4 C up on 5 D (9), 6 S up on 7 H (3), 4 S up on 5 H (7) = 62 moves in all. This is my record; perhaps the reader can beat it.

104. A TRICK WITH DICE.

All you have to do is to deduct 250 from the result given, and the three figures in the answer will be the three points thrown with the dice. Thus, in the throw we gave, the number given would be 386; and when we deduct 250 we get 136, from which we know that the throws were 1, 3, and 6.

The process merely consists in giving $100a + 10b + c + 250$, where a, b, and *c* represent the three throws. The result is obvious.

105. THE VILLAGE CRICKET MATCH.



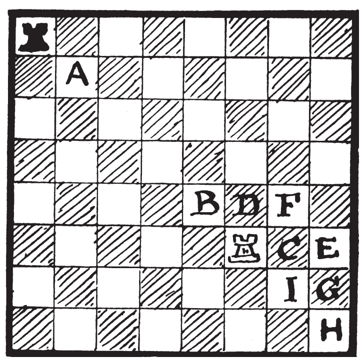
The diagram No. 1 will show that as neither Mr. Podder nor Mr. Dumkins can ever have been within the crease opposite to that from which he started, Mr. Dumkins would score nothing by his performance. Diagram No. 2 will, however, make it clear that since Mr. Luffey and Mr. Struggles have, notwithstanding their energetic but careless movements, contrived to change places, the manoeuvre must increase Mr. Struggles's total by one run.

106. THE PEBBLE GAME.

In the case of fifteen pebbles, the first player wins if he first takes two. Then when he holds an odd number and leaves 1, 8, or 9 he wins, and when he holds an even number and leaves 4, 5, or 12 he also wins. He can always do one or other of these things until the end of the game, and so defeat his opponent. In the case of thirteen pebbles the first player must lose if his opponent plays correctly. In fact, the only numbers with which the first player ought to lose are 5 and multiples of 8 added to 5, such as 13, 21, 29, etc.

107. THE TWO ROOKS.

The second player can always win, but to ensure his doing so he must always place his rook, at the start and on every subsequent move, on the same diagonal as his opponent's rook. He can then force his opponent into a corner and win. Supposing the diagram to represent the positions of the rooks at the start, then, if Black played first, White might have placed his rook at A and won next move. Any square on that diagonal from A to H will win, but the best play is always to restrict the moves of the opposing rook as much as possible. If White played first, then Black should have placed his rook at B (F would not be so good, as it gives White more scope); then if White goes to C, Black moves to D; White to E, Black to F; White to G, Black to C; White to H, Black to I; and Black must win next move. If at any time Black had failed to move on to the same diagonal as White, then White could take Black's diagonal and win.



108. PUSS IN THE CORNER.

No matter whether he plays first or second, the player A, who starts the game at 55, must win. Assuming that B adopts the very best lines of play in order to prolong as much as possible his existence, A, if he has first move, can always on his 12th move capture B; and if he has the second move, A can always on his 14th move make the capture. His point is always to get diagonally in line with his opponent, and by going to 33, if he has first move, he prevents B getting diagonally in line with himself. Here are two good games. The number in front of the hyphen is always A's move; that after the hyphen is B's:—

33-8, 32-15, 31-22, 30-21, 29-14, 22-7, 15-6, 14-2, 7-3, 6-4, 11-, and A must capture on his next (12th) move, -13, 54-20, 53-27, 52-34, 51-41, 50-34, 42-27, 35-20, 28-13, 21-6, 14-2, 7-3, 6-4, 11-, and A must capture on his next (14th) move.

109. A MATCH MYSTERY.

If you form the three heaps (and are therefore the second to draw), any one of the following thirteen groupings will give you a win if you play correctly: 15, 14, 1; 15, 13, 2; 15, 12, 3; 15, 11, 4; 15, 10, 5; 15, 9, 6; 15, 8, 7; 14, 13, 3; 14, 11, 5; 14, 9, 7; 13, 11, 6; 13, 10, 7; 12, 11, 7.

The beautiful general solution of this problem is as follows. Express the number in every heap in powers of 2, avoiding repetitions and remembering that $2^0 = 1$. Then if you so leave the matches to your opponent that there is an even number of every power, you can win. And if at the start you leave the powers even, you can always continue to do so throughout the game. Take, as example, the last grouping given above–12, 11, 7. Expressed in powers of 2 we have—

12	=	8	4	-	-
11	=	8	-	2	1
7	=	-	4	2	1
		2	2	2	2

As there are thus two of every power, you must win. Say your opponent takes 7 from the 12 heap. He then leaves—

5	=	-	4	-	1
11	=	8	-	2	1
7	=	-	4	2	1
		1	2	2	3

Here the powers are not all even in number, but by taking 9 from the 11 heap you immediately restore your winning position, thus—

5	=	-	4	-	1
2	=	-	-	2	-
7	=	-	4	2	1
		-	2	2	2

And so on to the end. This solution is quite general, and applies to any number of matches and any number of heaps. A correspondent informs me that this puzzle game was first propounded by Mr. W.M.F. Mellor, but when or where it was published I have not been able to ascertain.

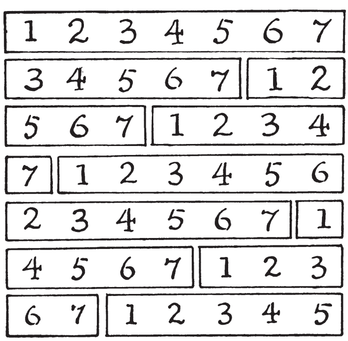
110. THE TROUBLESOME EIGHT.

4½	8	2½
3	5	7
7½	2	5½

The conditions were to place a different number in each of the nine cells so that the three rows, three columns, and two diagonals should each add up 15. Probably the reader at first set himself an impossible task through reading into these conditions something which is not there—a common error in puzzle-solving. If I had said "a different figure," instead of "a different number," it would have been quite impossible with the 8 placed anywhere but in a corner. And it would have been equally impossible if I had said "a different whole number." But a number may, of course, be fractional, and therein lies the secret of the puzzle. The arrangement shown in the figure will be found to comply exactly with the conditions: all the numbers are different, and the square adds up 15 in all the required eight ways.

111. THE MAGIC STRIPS.

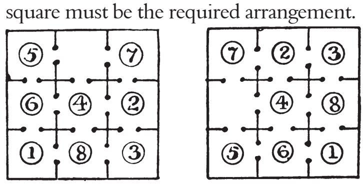
There are of course six different places between the seven figures in which a cut may be made, and the secret lies in keeping one strip intact and cutting each of the other six in a different place. After the cuts have been made there are a large number of ways in which the thirteen pieces may be placed together so as to form a magic square. Here is one of them:—



The arrangement has some rather interesting features. It will be seen that the uncut strip is at the top, but it will be found that if the bottom row of figures be placed at the top the numbers will still form a magic square, and that every successive removal from the bottom to the top (carrying the uncut strip stage by stage to the bottom) will produce the same result. If we imagine the numbers to be on seven complete *perpendicular* strips, it will be found that these columns could also be moved in succession from left to right or from right to left, each time producing a magic square.

112. EIGHT JOLLY GAOL-BIRDS.

There are eight ways of forming the magic square—all merely different aspects of one fundamental arrangement. Thus, if you give our first square a quarter turn you will get the second square; and as the four sides may be in turn brought to the top, there are four aspects. These four in turn reflected in a mirror produce the remaining four aspects. Now, of these eight arrangements only four can possibly be reached under the conditions, and only two of these four can be reached in the fewest possible moves, which is nineteen. These two arrangements are shown. Move the men in the following order: 5, 3, 2, 5, 7, 6, 4, 1, 5, 7, 6, 4, 1, 6, 4, 8, 3, 2, 7, and you get the first square. Move them thus: 4, 1, 2, 4, 1, 6, 7, 1, 5, 8, 1, 5, 6, 7, 5, 6, 4, 2, 7, and you have the arrangement in the second square. In the first case every man has moved, but in the second case the man numbered 3 has never left his cell. Therefore No. 3 must be the obstinate prisoner, and the second square must be the required arrangement.



113. NINE JOLLY GAOL BIRDS.

There is a pitfall set for the unwary in this little puzzle. At the start one man is allowed to be placed on the shoulders of another, so as to give always one empty cell to enable the prisoners to move about without any two ever being in a cell together. The two united prisoners are allowed to add their numbers together, and are, of course, permitted to remain together at the completion of the magic square. But they are obviously not compelled so to remain together, provided that one of the pair on his final move does not break the condition of entering a cell already occupied. After the acute solver has noticed this point, it is for him to determine which method is the better one—for the two to be together at the count or to separate. As a matter of fact, the puzzle can be solved in seventeen moves if the men are to remain together; but if they separate at the end, they may actually save a move and perform the feat in sixteen! The trick consists in placing the man in the centre on the back of one of the corner men, and then working the pair into the centre before their final separation.

A

2	9	4
7	5	3
6	1	8

B

6	7	2
1	5	9
8	3	4

Here are the moves for getting the men into one or other of the above two positions. The numbers are those of the men in the order in which they move into the cell that is for the time being vacant. The pair is shown in brackets:—

Place 5 on 1. Then, 6, 9, 8, 6, 4, (6), 2, 4, 9, 3, 4, 9, (6), 7, 6, 1.

Place 5 on 9. Then, 4, 1, 2, 4, 6, (14), 8, 6, 1, 7, 6, 1, (14), 3, 4,9.

Place 5 on 3. Then, 6, (8), 2, 6, 4, 7, 8, 4, 7, 1, 6, 7, (8), 9, 4, 3.

Place 5 on 7. Then, 4, (12), 8, 4, 6, 3, 2, 6, 3, 9, 4, 3, (12), 1, 6, 7.

The first and second solutions produce Diagram A; the second and third produce Diagram B. There are only sixteen moves in every case. Having found the fewest moves, we had to consider how we were to make the burdened man do as little work as possible. It will at once be seen that as the pair have to go into the centre before separating they must take at fewest two moves. The labour of the burdened man can only be reduced by adopting the other method of solution, which, however, forces us to take another move.

114. THE SPANISH DUNGEON.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

10	9	7	4
6	5	11	8
1	2	12	15
13	14		3

This can best be solved by working backwards—that is to say, you must first catch your square, and then work back to the original position. We must first construct those squares which are found to require the least amount of readjustment of the numbers. Many of these we know cannot possibly be reached. When we have before us the most favourable possible arrangements, it then becomes a question of careful analysis to discover which position can be reached in the fewest moves. I am afraid, however, it is only after considerable study and experience that the solver is able to get such a grasp of the various "areas of disturbance" and methods of circulation that his judgement is of much value to him.

The second diagram is a most favourable magic square position. It will be seen that prisoners 4, 8, 13, and 14 are left in their original cells. This position may be reached in as few as thirty-seven moves. Here are the moves: 15, 14, 10, 6, 7, 3, 2, 7, 6, 11, 3, 2, 7, 6, 11, 10, 14, 3, 2, 11, 10, 9, 5, 1, 6, 10, 9, 5, 1, 6, 10, 9, 5, 2, 12, 15, 3. This short solution will probably surprise many readers who may not find a way under from sixty to a hundred moves. The clever prisoner was No. 6, who in the original illustration will be seen with his arms extended calling out the moves. He and No. 10 did most of the work, each changing his cell five times. No. 12, the man with the crooked leg, was lame, and therefore fortunately had only to pass from his cell into the next one when his time came round.

115. CARD MAGIC SQUARES.

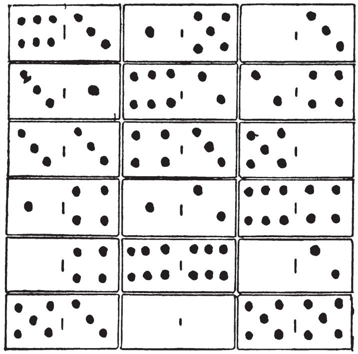
Arrange the cards as follows for the three new squares:—

3	2	4	6	5	7	9	8	10
4	3	2	7	6	5	10	9	8
2	4	3	5	7	6	8	10	9

Three aces and one ten are not used. The summations of the four squares are thus: 9, 15, 18, and 27–all different, as required.

116. THE EIGHTEEN DOMINOES.

The illustration explains itself. It will be found that the pips in every column, row, and long diagonal add up 18, as required.



117. TWO NEW MAGIC SQUARES.

Here are two solutions that fulfil the conditions:—

SUBTRACTING

11	4	14	13
16	7	1	2
6	5	3	12
9	10	8	15

DIVIDING

36	8	54	27
216	12	1	2
6	3	4	72
9	18	24	108

The first, by subtracting, has a constant 8, and the associated pairs all have a difference of 4. The second square, by dividing, has a constant 9, and all the associated pairs produce 3 by division. These are two remarkable and instructive squares.

118. MAGIC SQUARES OF TWO DEGREES.

The following is the square that I constructed. As it stands the constant is 260. If for every number you substitute, in its allotted place, its square, then the constant will be 11,180. Readers can write out for themselves the second degree square.

The main key to the solution is the pretty law that if eight numbers sum to

7	53	41	27	2	52	48	30
12	58	38	24	13	63	35	17
51	1	29	47	54	8	28	42
64	14	18	36	57	11	23	37
25	43	55	5	32	46	50	4
22	40	60	10	19	33	61	15
45	31	3	49	44	26	6	56
34	20	16	62	39	21	9	59

260 and their squares to 11,180, then the same will happen in the case of the eight numbers that are complementary to 65. Thus 1 + 18 + 23 26 + 31 + 48 + 56 + 57 = 260, and the sum of their squares is 11,180. Therefore 64 + 47 + 42 + 39 + 34 + 17 + 9 + 8 (obtained by subtracting each of the above numbers from 65) will sum to 260 and their squares to 11,180. Note that in every one of the sixteen smaller squares the two diagonals sum to 65. There are four columns and four rows with their complementary columns and rows. Let us pick out the numbers found in the 2nd, 1st, 4th, and 3rd rows and arrange them thus :—

1	8	28	29	42	47	51	54
2	7	27	30	41	48	52	53
3	6	26	31	44	45	49	56
4	5	25	32	43	46	50	55

Here each column contains four consecutive numbers cyclically arranged, four running in one direction and four in the other. The numbers in the 2nd, 5th, 3rd, and 8th columns of the square may be similarly grouped. The great difficulty lies in discovering the conditions governing these groups of numbers, the pairing of the complementaries in the squares of four and the formation of the diagonals. But when a correct solution is shown, as above, it discloses all the more important keys to the mystery. I am inclined to think this square of two degrees the most elegant thing that exists in magics. I believe such a magic square cannot be constructed in the case of any order lower than 8.

119. WHO WAS FIRST?

Biggs, who saw the smoke, would be first; Carpenter, who saw the bullet strike the water, would be second; and Anderson, who heard the report, would be last of all.

120. A WONDERFUL VILLAGE.

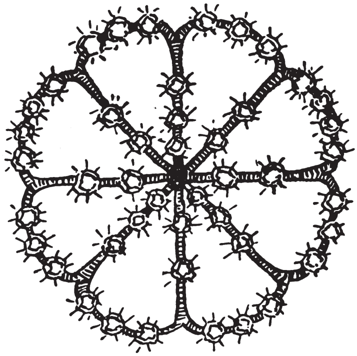
When the sun is in the horizon of any place (whether in Japan or elsewhere), he is the length of half the earth's diameter more distant from that place than in his meridian at noon. As the earth's semi-diameter is nearly 4,000 miles, the sun must be considerably more than 3,000 miles nearer at noon than at his rising, there being no valley even the hundredth part of 1,000 miles deep.

121. A CALENDAR PUZZLE.

The first day of a century can never fall on a Sunday; nor on a Wednesday or a Friday.

122. THE RUBY BROOCH.

In this case we were shown a sketch of the brooch exactly as it appeared after the four rubies had been stolen from it. The reader was asked to show the positions from which the stones "may have been taken;" for it is not possible to show precisely how the gems were originally placed, because there are many such ways. But an important point was the statement by Lady Littlewood's brother: "I know the brooch well. It originally



contained forty-five stones, and there are now only forty-one. Somebody has stolen four rubies, and then reset as small a number as possible in such a way that there shall always be eight stones in any of the directions you have mentioned."

The diagram shows the arrangement before the robbery. It will be seen that it was only necessary to reset one ruby—the one in the centre. Any solution involving the resetting of more than one stone is not in accordance with the brother's statement, and must therefore be wrong. The original arrangement was, of course, a little unsymmetrical, and for this reason the brooch was described as "rather eccentric."

123. PHEASANT-SHOOTING.

There were 24 pheasants at the start. Of these 16 were shot dead, 1 was wounded in the wing, and 7 got away but "how many still remained?" Now the poor bird that was wounded in the wing, though unable to fly, was very active in its painful struggles to run away. The answer is, therefore, that the 16 birds that were shot dead "still remained," or "remained still."

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